

Polar Coding Assisted Diversity Based Multiple-Input Multiple-Output (MIMO) Scheme

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Abbreviations

AWGN - Additive white Gaussian noise

BER – Bit Error Rate

DSTBC - Differential Space-time block codes

MIMO - Multiple-Input Multiple-Output

ML – Maximum Likelihood

SNR - Signal-To-Noise Ratio

STBC - space-time block codes

Week one (23/06-29/06)

Diversity

It is important to provide an architecture with diversity to maintain high performance in wireless mobile communications. Achieving diversity can be done by using multiple antennas, using interleaved coded modulation, resolving propagation paths in time or spatially, and using multicarrier transmission. In interleaved coded modulation, which is the most widely used, the redundancy is added for providing diversity, and the interleaving separates the code symbols providing independent fading distortion for each of the code symbols. For the standard interleaved coded modulation, the challenge is a tradeoff must be made between decoding delay and demodulation performance. Transmission diversity is an effective technology for combating fading in wireless communications. Its strength is diversity can be achieved without greatly increasing the complexity of the receiver by transmitting from multiple spatially separated antennas. [1]

Differential Space-time block codes

A system equipped with two transmit antennas and an M-PSK modulation set $A = \{e^{\frac{2\pi j k}{M}}/\sqrt{2}, k = 0, \dots, M - 1\}$, where $M = 2^b$. The transmitter begins with sending an arbitrary initial signal vector $S_0 = (s_1, s_2)$, which is unknown to the receiver and does not carry any information. After the STBC encoding, s_1 and s_2 are transmitted by two antennas at time 1 and $-s_2$ and s_1 are transmitted at time 2. Assume that $S_{t-1} = (s_{2t-1}, s_{2t})$ is the signal vector fed to the STBC encoder in the $(t - 1)$ encoding operation. Two output signal vectors are generated by encoder are (s_{2t-1}, s_{2t}) and $(-s_{2t}^*, s_{2t-1}^*)$, an orthonormal basis is formed and can be denoted by

$$D(S_{t-1}) = \begin{bmatrix} s_{2t-1} & s_{2t} \\ -s_{2t}^* & s_{2t-1}^* \end{bmatrix}.$$

$D(S_{t-1})$ is the reference to calculate the next STBC encoder input $S_t = (s_{2t+1}, s_{2t+2})$ as

$$S_t = R_t D(S_{t-1}),$$

where $R_t = (R_t^1, R_t^2)$ is the differential coefficient vector. The equation above is the differential encoding rule for DSTBC. This process is inductively repeated until the end of the frame.

The mapping M from the information blocks to R is defined in following. Assume information block $C_t \approx (c_t^1, c_t^2)$ reach in the t -th encoding operation, where c_t^1 and c_t^2 consist of b bits each. C_t is Gray-mapped into two M-PSK symbols $X_t(C_1) = (x_t^1, x_t^2) \in A \times A$. The corresponding differential coefficient vector $R_t(G)$ is computed to satisfy equation $X_t(C_t) = R_t C_t D(X_0)$, where $X_0 = (\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}})$ [2].

Week two (30/06-06/07)

Space-Time Block Codes

Space-time block code (STBC) describes the relationship between the original transmitted signal x and the signal replicas artificially created at the transmitter for transmission over various diversity channels, and it is expressed as an $n \times p$ dimensional transmission matrix, where the number of transmitter antennas is p and the number of time slots used to transmit k input symbols is n . The entries of the matrix are constituted by linear combinations of the k -ary input symbols x_1, x_2, \dots, x_k representing the information-bearing binary bits to be transmitted over the transmit diversity channels, and their conjugates. A block of $k \times b$ binary bits are entered into the space-time block encoder at a time and it is therefore referred to as a space-time block code. Therefore, the transmission matrix of a STBC can be expressed generally as:

$$\begin{pmatrix} g_{11} & g_{21} & \cdots & g_{p1} \\ g_{12} & g_{22} & \cdots & g_{p2} \\ \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot \\ g_{1n} & g_{2n} & \cdots & g_{pn} \end{pmatrix}, \quad (1)$$

where the entries g_{ij} represent linear combinations of the symbols x_1, x_2, \dots, x_k and their conjugates. It is names space-time coding because it is carried out in both space and time.

To be more exact, if there is a twin-transmitter based STBC, the transmission matrix is defined as follows:

$$G_2 = \begin{pmatrix} x_1 & x_2 \\ -\bar{x}_2 & \bar{x}_1 \end{pmatrix}. \quad (2)$$

If one receiver is used, the received signal is given by:

$$y_1 = h_1 x_1 + h_2 x_2 + n_1 \quad (3)$$

$$y_2 = -h_1 \bar{x}_2 + h_2 \bar{x}_1 + n_2 \quad (4)$$

With the help of combiner and channel estimator, the extracted signal x_1 can be expressed as:

$$\begin{aligned} \tilde{x}_1 &= \bar{h}_1 y_1 + h_2 \bar{y}_2 \\ &= \bar{h}_1 h_1 x_1 + \bar{h}_1 h_2 x_2 + \bar{h}_1 n_1 - h_2 \bar{h}_1 x_2 + h_2 \bar{h}_2 x_1 + h_2 \bar{n}_2 \\ &= (|h_1|^2 + |h_2|^2) x_1 + \bar{h}_1 n_1 + h_2 \bar{n}_2. \end{aligned} \quad (5)$$

And the signal x_2 generated is:

$$\begin{aligned} \tilde{x}_2 &= \bar{h}_2 y_1 - h_1 \bar{y}_2 \\ &= \bar{h}_2 h_1 x_1 + \bar{h}_2 h_2 x_2 + \bar{h}_2 n_1 + h_1 \bar{h}_1 x_2 - h_1 \bar{h}_2 x_1 - h_1 \bar{n}_2 \end{aligned}$$

$$= (|h_1|^2 + |h_2|^2)x_2 + \bar{h}_2 n_1 - h_2 \bar{n}_2. \quad (6)$$

combine the received signals y_1 and y_2 as follows:

$$\begin{aligned} \tilde{x}_1 &= \bar{h}_1 y_1 + h_2 \bar{y}_2 \\ &= \bar{h}_1 h_1 x_1 + \bar{h}_1 h_2 x_2 + \bar{h}_1 n_1 - h_2 \bar{h}_1 x_2 + h_2 \bar{h}_2 \bar{x}_1 + h_2 \bar{n}_2 \\ &= (|h_1|^2 + |h_2|^2) x_1 + \bar{h}_1 n_1 + h_2 \bar{n}_2. \end{aligned} \quad (9.15)$$

Similarly, for signal x_2 we generate:

$$\begin{aligned} \tilde{x}_2 &= \bar{h}_2 y_1 - h_1 \bar{y}_2 \\ &= \bar{h}_2 h_1 x_1 + \bar{h}_2 h_2 x_2 + \bar{h}_2 n_1 + h_1 \bar{h}_1 x_2 - h_1 \bar{h}_2 x_1 - h_1 \bar{n}_2 \\ &= (|h_1|^2 + |h_2|^2) x_2 + \bar{h}_2 n_1 - h_1 \bar{n}_2. \end{aligned} \quad (9.16)$$

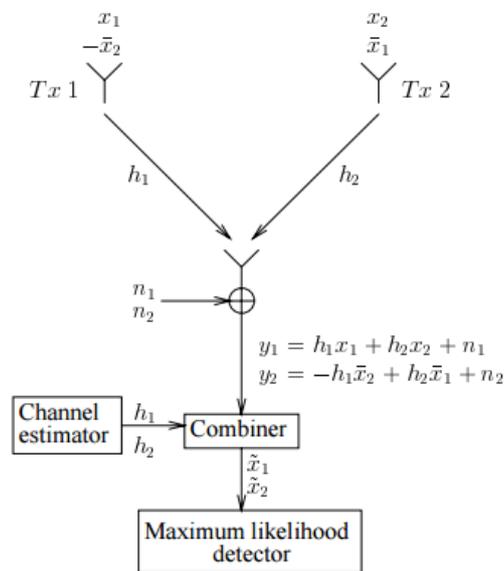


Figure 1. Baseband representation of the simple twin-transmitter space-time block code G2 of Equation 2 using one receiver [3].

Week three (07/07-13/07)

Simulation of STBC

MATLAB Code:

```
clear

% antenna
Nt = 2;
Nr = 1;

% SNR range
SNR_dB = -10:2:40;
BER = zeros(1,length(SNR_dB)); %BER

% Modulation scheme
modem_name1 = 'BPSK'
```

```

modulation = [+1, -1];

%estimated symbol vector
Symbol_index = [1,1;1,-1;-1,1;-1,-1]'; %4 possible combination
[~, Symbol_Searching] = size(Symbol_index);

for i = 1:length(SNR_dB)
    errors = 0;%total errors
    total_bits = 0;%total bit when these errors generated

    %set the maximum of errors
    if SNR_dB(i) < 10
        errors_limit = 8000;
    elseif SNR_dB(i) < 20
        errors_limit = 1000;
    elseif SNR_dB(i) < 30
        errors_limit = 600;
    else
        errors_limit = 200;
    end
    while errors < errors_limit
% Channel
% Uncorrelated Rayleigh fading channel
channel = sqrt(1/2)*(randn(1,2)+j*randn(1,2));
channel = [channel;-conj(channel(2)) conj(channel(1))];

% AWGN channel with no fading
% channel = ones(1,symbol_count) + i*zeros(1,symbol_count);

% Generate some random symbols
symbols = ceil(length(modulation)*rand(1,2));

% Generate the transmitted signal
x = modulation(symbols)';

% Generate some noise
N0 = 1/(10^(SNR_dB(i)/10));
noise = sqrt(N0/2)*(randn(2,1)+j*randn(2,1));

% Generate the received signal
r = channel*x+noise;

% z = channel'*r;

    %% demodulation

    %ML detector
    x_hat = Symbol_index(:, 1);
    for j = 1:Symbol_Searching
        Symbol = Symbol_index(:,j);
        if norm(r - channel*Symbol) < norm(r -
channel*x_hat) %?y-Hx?^2
            x_hat = Symbol;
        end
    end
end

```

```

%       x_hat = [real(x_hat) > 0 imag(x_hat) > 0]; %demapping
errors = errors + sum(x ~= x_hat) ; %record the number of
errors
total_bits = total_bits + length(x); %record the number of
bits
end

BER(i) = errors/total_bits; %compute the bit error correspondinng
to the SNR
end

save('BER.mat', 'SNR_dB', 'BER')
figure
semilogy(SNR_dB,BER);
grid on
title('BER verse SNR for STBC')
legend('BPSK')

```

result:

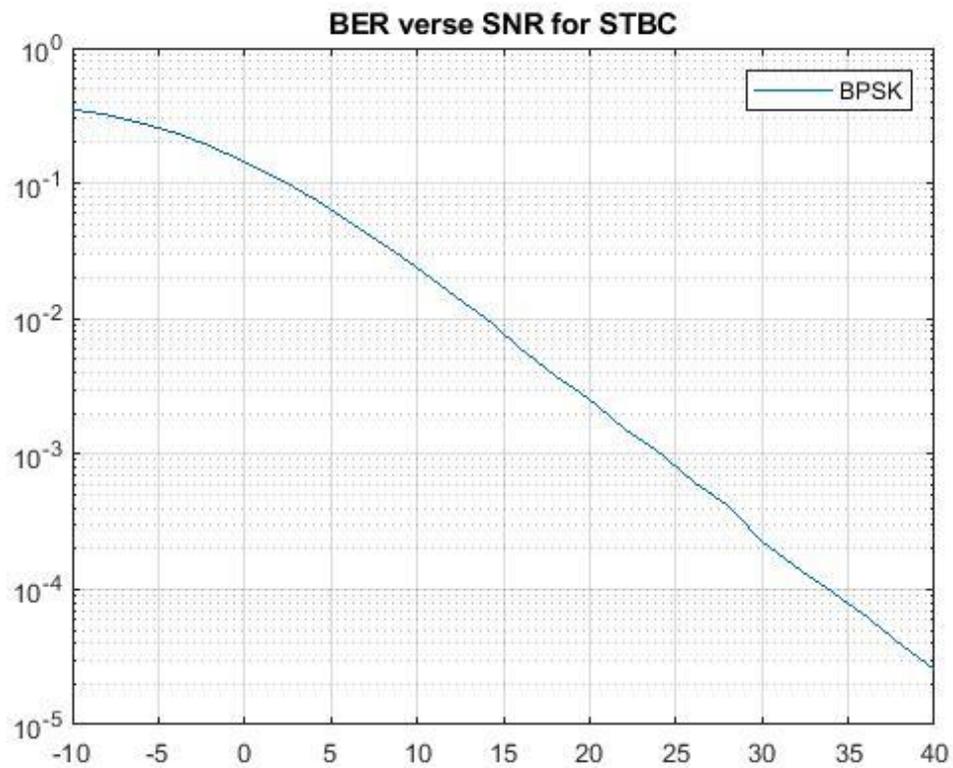


Figure 2. BER versus SNR for STBC

Channel capacity:

Result:

The channel capacity of different modulation scheme over AWGN channel

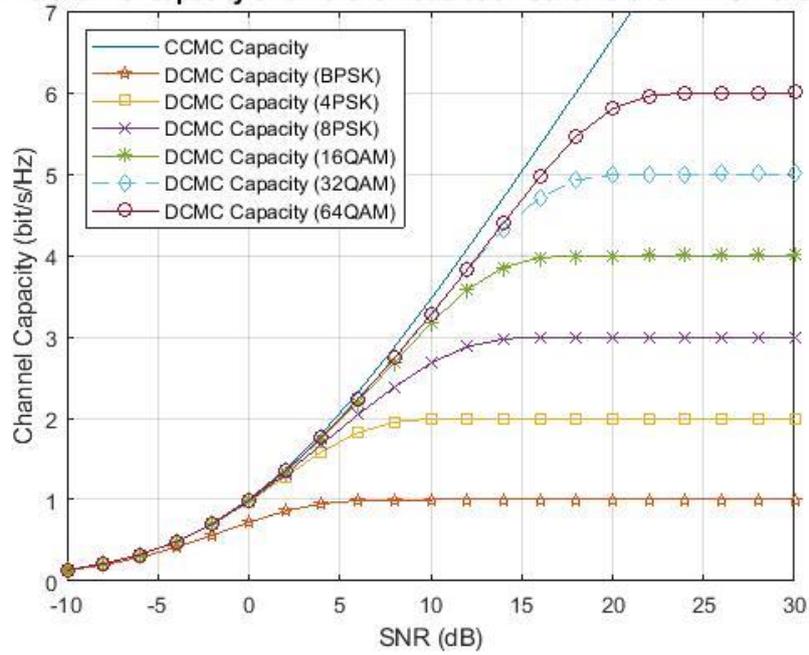


Figure 3. channel capacity over AWGN channel

The channel capacity of different modulation scheme over uncorrelated Rayleigh channel

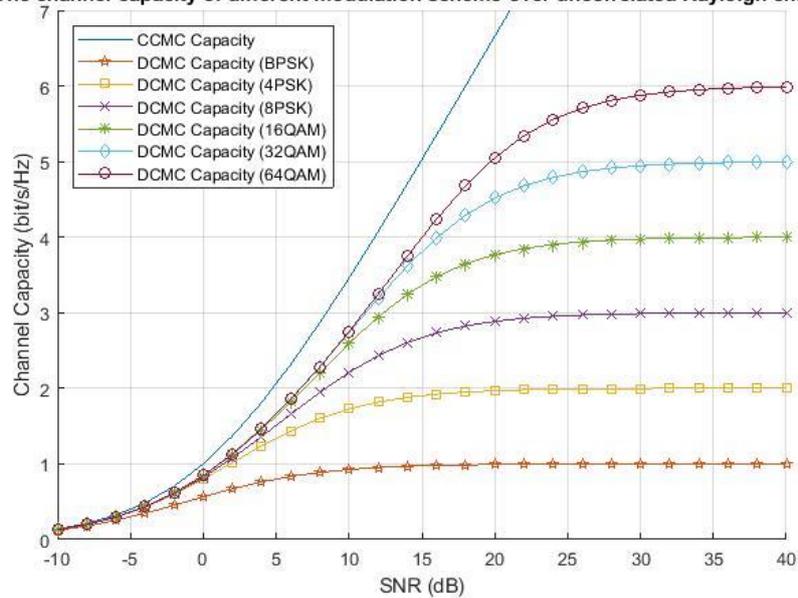


Figure 4. channel capacity over Rayleigh channel

Week Four (14/07-20/07)

Updated simulation of STBC

Code:

```
clear
```

```
% antenna
```

```
Nt = 2;
```

```

Nr = 1;

% SNR range
SNR_dB = 0:40;
BER_BPSK = zeros(1,length(SNR_dB)); %BER

% Modulation scheme

modem_name1 = 'BPSK'
modulation = [+1, -1];

%estimated symbol vector
Symbol_index = [1,-1]; %2 possible value
[~, Symbol_Searching] = size(Symbol_index);

for i = 1:length(SNR_dB)
    errors = 0;%total errors
    total_bits = 0;%total bit when these errors generated

    %set the maximum of errors
    if SNR_dB(i) < 10
        errors_limit = 10000;
    elseif SNR_dB(i) < 20
        errors_limit = 3000;
    elseif SNR_dB(i) < 30
        errors_limit = 2000;
    else
        errors_limit = 1000;
    end
    while errors < errors_limit
% Channel
% Uncorrelated Rayleigh fading channel
% channel = sqrt(1/2)*(randn(1,2)+j*randn(1,2));
% channel = [channel;-conj(channel(2)) conj(channel(1))];
channel = sqrt(1/2)*(randn(1,2)+j*randn(1,2));

% AWGN channel with no fading
% channel = ones(1,symbol_count) + i*zeros(1,symbol_count);

% Generate some random symbols
symbols = ceil(length(modulation)*rand(2,1));

% Generate the transmitted signal
x = modulation(symbols)';
% G = [x';-conj(x(2)), conj(x(1))];

% Generate some noise
N0 = 1/(10^(SNR_dB(i)/10));
noise = sqrt(N0/2)*(randn(2,1)+j*randn(2,1));

% Generate the received signal
H = sqrt(1/2)*[channel; conj(channel(2)) -conj(channel(1))];
r = H*x+noise;
% y = [r(1);conj(r(2))];
% x_hat=H*y;
z = H'*r;
A = H'*H;

```

```

%% demodulation
a = A(1,1);
%ML detector
x_hat = zeros(2,1);
x_hat_1 = Symbol_index(:,1);
for m = 1:Nt
    for j = 1:Symbol_Searching
        Symbol = Symbol_index(:,j);
        if norm(z(m) - a*Symbol) < norm(z(m) - a*x_hat_1)
            x_hat_1 = Symbol;
        end
    end
    x_hat(m)=x_hat_1;
end

%       x_hat = [real(x_hat) > 0 imag(x_hat) > 0]; %demapping
errors = errors + sum(x ~= x_hat) ; %record the number of
errors
total_bits = total_bits + length(x); %record the number of
bits
end

BER_BPSK(i) = errors/total_bits; %compute the bit error
corresponding to the SNR
end

save('BER_BPSK.mat','SNR_dB','BER_BPSK')
figure
semilogy(SNR_dB,BER_BPSK);
grid on
title('BER verse SNR for STBC')
legend('BPSK')

```

Result:

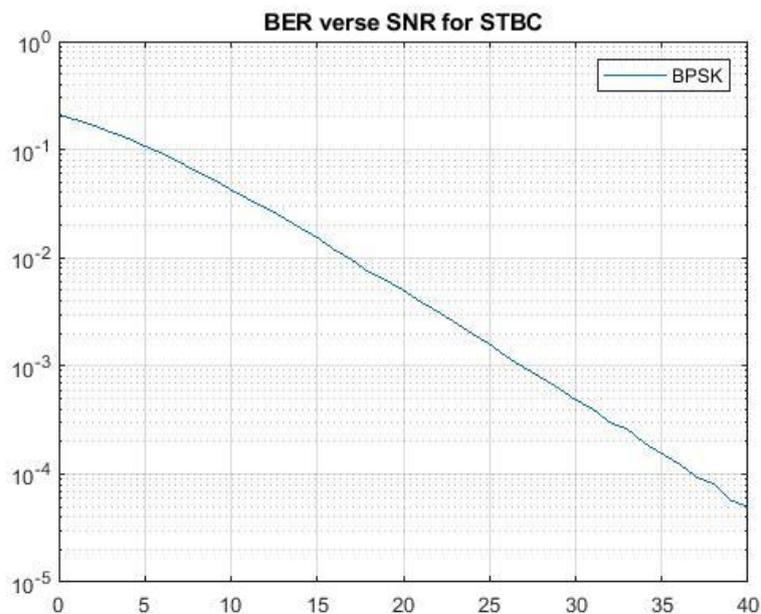


Figure 5. BER versus SNR for STBC

Updated channel capacity

Compared with the previous simulation of channel capacity, the axis range of the plot for Additive white Gaussian noise (AWGN) channel was set as $[-10 \ 40 \ 0 \ 7]$ which is same to that of the plot for Rayleigh channel.

Result:

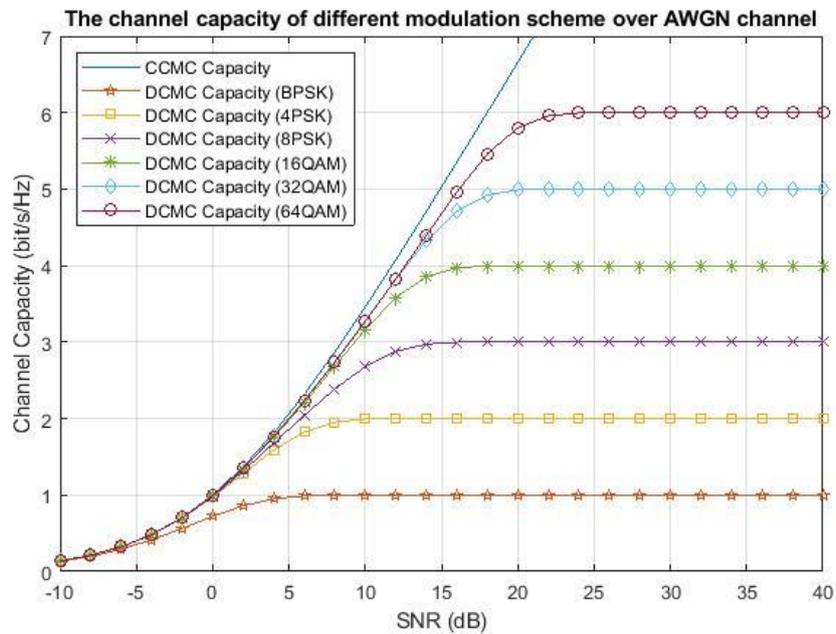


Figure 6. Channel capacity over AWGN channel

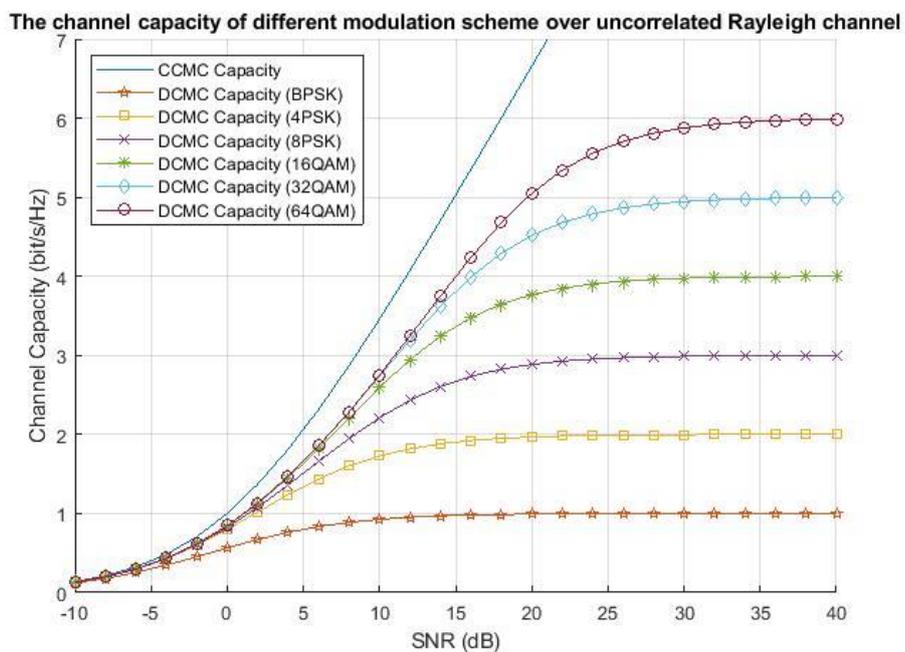


Figure 7. Channel capacity over Rayleigh channel

Week Five (21/07-27/07)

Updated simulation of STBC

This simulation version follows step by step method. To be more exact, assume that there are two transmit antennas ($M = 2$) employed, hence, the transmission matrix of STBC can be expressed as:

$$G_2 = \frac{1}{\sqrt{2}} \begin{bmatrix} x_1 & x_2 \\ -x_2^* & x_1^* \end{bmatrix}, \quad (1)$$

where x_1 and x_2 are two consecutive symbols transmitted. The observations across two time slots at the n th receive antenna is given by:

$$\begin{bmatrix} y_{1n} \\ y_{2n} \end{bmatrix} = \frac{1}{\sqrt{2}} \begin{bmatrix} x_1 & x_2 \\ -x_2^* & x_1^* \end{bmatrix} \begin{bmatrix} h_{1n} \\ h_{2n} \end{bmatrix} + \begin{bmatrix} n_{1n} \\ n_{2n} \end{bmatrix} \quad (2)$$

Then take the conjugate on y_{2n} in Eq. (2) to have

$$r_n = \begin{bmatrix} y_{1n} \\ y_{2n}^* \end{bmatrix} = \frac{1}{\sqrt{2}} \underbrace{\begin{bmatrix} h_{1n} & h_{2n} \\ h_{2n}^* & -h_{1n}^* \end{bmatrix}}_{\mathbf{H}_n} \underbrace{\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}}_{\mathbf{x}} + \underbrace{\begin{bmatrix} n_{1n} \\ n_{2n}^* \end{bmatrix}}_{\mathbf{n}_n}. \quad (3)$$

The received signal associated with two consecutive symbol intervals can be expressed as following when the number of receive antennas is N ,

$$\mathbf{r} = \mathbf{H}\mathbf{x} + \mathbf{n} \quad (4)$$

where

$$\mathbf{r} = [r_1^T \quad r_2^T \quad \dots \quad r_N^T]^T, \quad (2N \times 1) \quad (5)$$

$$\mathbf{H} = [\mathbf{H}_1^T \quad \mathbf{H}_2^T \quad \dots \quad \mathbf{H}_N^T]^T, \quad (2N \times M) \quad (6)$$

$$\mathbf{n} = [n_1^T \quad n_2^T \quad \dots \quad n_N^T]^T. \quad (2N \times 1) \quad (7)$$

To obtain the decision variable for \mathbf{x} , \mathbf{H}^H has to multiply \mathbf{r} :

$$\mathbf{z} = \mathbf{H}^H \mathbf{r} = \frac{1}{2} \underbrace{\sum_{n=1}^N (|h_{1n}|^2 + |h_{2n}|^2)}_{\mathbf{A}} \mathbf{x} + \mathbf{H}^H \mathbf{n}, \quad (8)$$

because

$$\begin{aligned} \mathbf{H}^H \mathbf{H} &= \frac{1}{2} \begin{bmatrix} |h_{1n}|^2 + |h_{2n}|^2 & 0 \\ 0 & |h_{1n}|^2 + |h_{2n}|^2 \end{bmatrix} \\ &= \frac{|h_{1n}|^2 + |h_{2n}|^2}{2} \mathbf{I}_2 \end{aligned} \quad (10)$$

Then, maximum likelihood (ML) detection is used to recover transmit symbols one by one:

$$\hat{\mathbf{x}}_m = \arg \min_{\mathbf{x}} \|\mathbf{z}_m - \mathbf{A}(1,1)\mathbf{x}\|^2, \quad (11)$$

where \mathbf{A} is equal to $\mathbf{H}^H \mathbf{H}$, $\mathbf{A}(1,1)$ can be used for detecting all symbols because $\mathbf{A}(1,1)$ equals to $\mathbf{A}(2,2)$ according to Eq. (10).

The plot below shows the relationship between bit error rate (BER) and signal-to-noise ratio (SNR) for BPSK, QPSK, 8PSK and 16QAM.

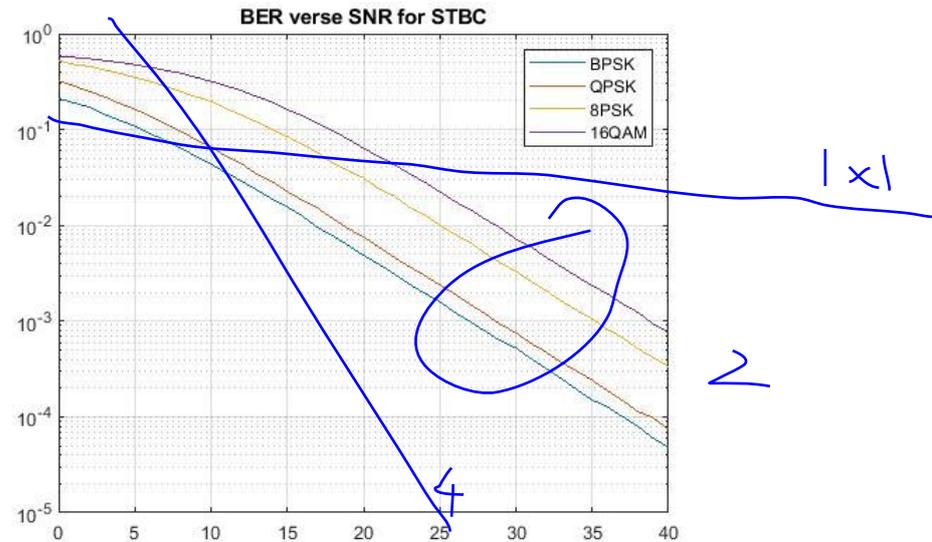


Figure 8. BER versus SNR for STBC

Codes for BPSK:

```
clear

% antenna
Nt = 2;
Nr = 1;

% SNR range
SNR_dB = 0:40;
BER_BPSK = zeros(1,length(SNR_dB)); %BER

% Modulation scheme

modem_name1 = 'BPSK'
modulation = [+1, -1];

%estimated symbol vector
Symbol_index = [1,-1]; %2 possible value
[~, Symbol_Searching] = size(Symbol_index);

for i = 1:length(SNR_dB)
    errors = 0;%total errors
    total_bits = 0;%total bit when these errors generated

    %set the maximum of errors
    if SNR_dB(i) < 10
        errors_limit = 10000;
    elseif SNR_dB(i) < 20
        errors_limit = 5000;
    elseif SNR_dB(i) < 30
        errors_limit = 3000;
    else
        errors_limit = 1500;
    end
end
```

```

    while errors < errors_limit
    % Channel
    % Uncorrelated Rayleigh fading channel
    % channel = sqrt(1/2)*(randn(1,2)+j*randn(1,2));
    % channel = [channel;-conj(channel(2)) conj(channel(1))];
    channel = sqrt(1/2)*(randn(2,1)+j*randn(2,1));

    % AWGN channel with no fading
    % channel = ones(1,symbol_count) + i*zeros(1,symbol_count);

    % Generate some random symbols
    symbols = ceil(length(modulation)*rand(2,1));

    % Generate the transmitted signal
    x = modulation(symbols);
    G = sqrt(1/2)*[x;-conj(x(2)), conj(x(1))];

    % Generate some noise
    N0 = 1/(10^(SNR_dB(i)/10));
    noise = sqrt(N0/2)*(randn(2,1)+j*randn(2,1));

    % Generate the received signal
    % r = H*x+noise;
    y = G * channel + noise;
    r = [y(1);conj(y(2))];
    H = sqrt(1/2)*[channel'; conj(channel(2)) -conj(channel(1))];
    z = H'*r;
    A = H'*H;

    %% demodulation
    a = A(1,1);
    %ML detector
    x_hat = zeros(1,2);
    x_hat_1 = Symbol_index(:,1);
    for m = 1:Nt
        for j = 1:Symbol_Searching
            Symbol = Symbol_index(:,j);
            if norm(z(m) - a*Symbol) < norm(z(m) - a*x_hat_1)
                x_hat_1 = Symbol;
            end
        end
        x_hat(m)=x_hat_1;
    end

    % x_hat = [real(x_hat) > 0 imag(x_hat) > 0]; %demapping
    errors = errors + sum(x ~= x_hat) ; %record the number of
errors
    total_bits = total_bits + length(x); %record the number of
bits
    end

    BER_BPSK(i) = errors/total_bits; %compute the bit error
corresponding to the SNR
end

save('BER_BPSK.mat','SNR_dB','BER_BPSK')
figure
semilogy(SNR_dB,BER_BPSK);
grid on

```

```

title('BER verse SNR for STBC')
legend('BPSK')

```

Polar codes

The polar codes with a clear construction invented by Arikan, are linear block error correcting codes, which has the ability to approach Shannon's capacity. In addition, polar codes' encoding and decoding complexity is low. Therefore, polar coding has been widely used in the case of multiple-access channels, wiretap channels, data compression due to its benefits, it also be employed to improve the channel performance in 5G field [4].

In IT++, command “bin/a.out Training=1 Rayleigh=0 FrameCount=1000 N=2048 K=1024 SNR=-4” is used in training. To obtain BER values, command “bin/a.out Training=0 Rayleigh=1 N=2048 K=1024 SNR=14 SNRStart=5 SNRDelta=1 SNRStop=50 TargetErrorCount=100 FrameCount=10000 ConfigFile=polar_2048_1024_Ray_T1R1_<modulation scheme>” are employed. Fig. 9 shows the two curves obtained, which represents the relationship between BER and SNR for BPSK and 4QAM.

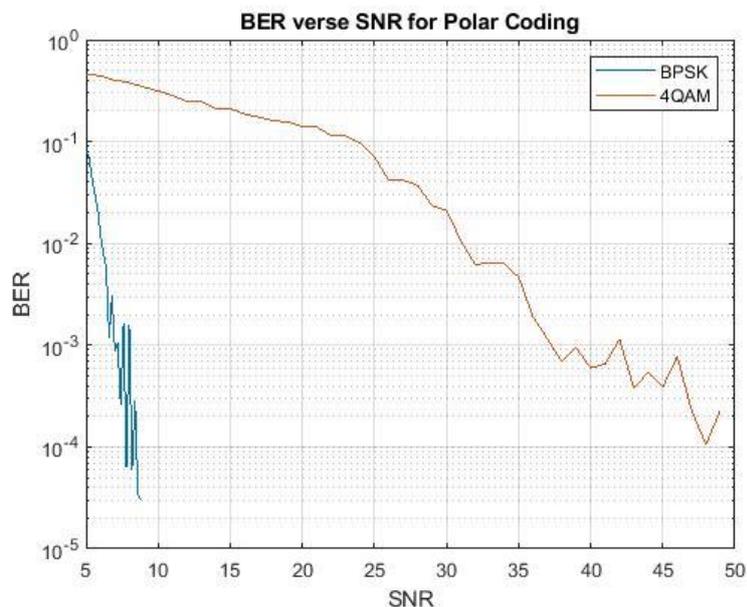


Figure 9. BER verse SNR for Polar Coding

Reference

- [1] Jiann-Ching Guey, M. P. Fitz, M. R. Bell and Wen-Yi Kuo, "Signal design for transmitter diversity wireless communication systems over Rayleigh fading channels," in *IEEE Transactions on Communications*, vol. 47, no. 4, pp. 527-537, April 1999, doi: 10.1109/26.764926.
- [2] J. Geng, U. Mitra and M. P. Fitz, "Optimal space-time block codes for CDMA systems," *MILCOM 2000 Proceedings. 21st Century Military Communications. Architectures and Technologies for Information Superiority (Cat. No.00CH37155)*, Los Angeles, CA, 2000, pp. 387-391 vol.1, doi: 10.1109/MILCOM.2000.904981.
- [3] L. Hanzo, T. H. Liew, B. L. Yeap, R.Y.S. Tee, S. X. Ng, "Turbo Coding, Turbo Equalisation and Space Time Coding: EXIT-Chart-Aided Near-Capacity Designs for Wireless Channels", New York, USA: John Wiley, IEEE Press, 2011.

[4] Tal, Ido, and Alexander Vardy. "How to construct polar codes." *IEEE Transactions on Information Theory*, vol. 59, no. 10, pp. 6562-6582, Oct. 2013.