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Modelling the una-corda effect in pianos

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Abstract. Most notes of a piano are fitted with two or three strings that, in normal playing conditions, are simultaneously struck by the hammer. In grand pianos, the *una corda* pedal offers alternative musical effects, notably a softer and duller tonal quality. This pedal's operation is linked to a lever system that displaces the action to one side causing the hammer to hit only one-out-of-two or two-out-of-three strings, thereby altering the sound. Meanwhile, the other strings of the same note undergo sympathetic vibration, owing to their structural connection through the bridge. This paper introduces a dynamic model that replicates the effects of the *una corda* pedal. It consists of a state-space scheme, serving as a framework to couple the dynamic behaviour of the various components. Stiff-string models describe the strings' vibration in three dimensions while a reduced modal model captures the dynamics of the soundboard. Results show how the string vibration and the force transmitted to the soundboard are affected by applying hammer excitation to individual or multiple strings. When all strings are struck, the transverse vibration shows beating. This effect is audible and evidenced in the spectrograms, where the amplitude of partials oscillates over time. In the *una corda* case, the force exerted on the bridge by the passive string increases initially with time, and its contribution to the overall transmitted force is smaller. However, there is no beating, the decay is more even, and the spectrograms do not show irregularities over time.

1. Introduction

The coupling between the strings of musical instruments is a physical phenomenon that defines the tonal characteristics, and it occurs due to the strings' connection with the instrument's bridge and soundboard. The connection with the soundboard can lead to the generation of vibration in the string in directions different from the excitation. This is commonly referred to as double polarization. The connection via the bridge can also generate vibration in non-excited strings that are left to vibrate freely. Sympathetic vibration of strings, can be controlled to some extent in pianos using the *una corda* pedal, which shifts the piano action mechanism to avoid striking one string of the duplet or triplet strings. The unstruck string is then free to vibrate and, due to the connection with the soundboard, is affected by the vibration of the struck strings.

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The coupling between piano strings and its effects was first noted by Weinreich [1], who also studied the effects of the *una corda* pedal. When the hammer strikes all the strings of the unison, due to small differences in the excitation and due to the mistuning between the strings, a longer decay rate - referred to by Weinreich as "aftersound"- of the double decay is generated. When the pedal is used, the resultant decay rate is longer relative to the attack (the peak due to the hammer strike). Weinreich performed this analysis numerically and experimentally focusing on a single string mode. Similar results have been observed in more recent research [2], where a psychoacoustic study was performed showing that when the pedal is not used the tone has higher amplitudes initially, while in the aftersound it has a smaller amplitude than when using the *una corda* pedal.

This work focuses on the effect of the generated sympathetic vibration in the unstruck string in comparison with the vibration generated if this string is struck. It considers two strings which are fully coupled in three directions due to the connection with the soundboard. To represent the *una corda*, the hammer is allowed to strike one string while the other one vibrates sympathetically due to the transmission of vibration through the bridge. The restriction of the model to two strings is representative of notes in the lower range, the modelling approach can be extended to notes in the treble range which normally include three strings, with the hammer striking two of them when the *una corda* pedal is depressed. The dynamic behaviour of the soundboard at the bridge is represented by a reduced model based on finite elements. The whole system is implemented in a state-space formulation. Several string modes are included to calculate the onset and decay of a piano tone and evaluate how this is influenced by the generation of sympathetic vibration.

2. String and soundboard

Two identical C2 strings of length L, differing only in tension, are connected to a soundboard system by means of contact springs and dampers, as shown in Figure 1. The figure also displays the main variables of the problem, namely the forces F at the hammer striking point e, located at a distance L_e from the agraffe termination, and at the connection points between the strings and soundboard (points s and b in figure). The connection with the soundboard divides the strings into two parts, the speaking length L_s and the remaining vibrating duplex scaling segment L_d, which can be muted in the model by a continuous distribution of viscous dampers. The soundboard consists of a reduced modal model which is also visually represented in the figure.

2.1. String models

A stiff string model is adopted to represent the two transverse motions of the strings in the directions transverse (T) and parallel (P) to the soundboard. The equations of motion refer to strings of length L, therefore disconnected from the soundboard, which are simply supported at the two ends defined by the agraffe and hitch pin (see Figure 1). The equations of motion are [3]:

$$\mu \frac{\partial^2 y_i}{\partial t^2} = T_{0,i} \frac{\partial^2 y_i}{\partial x^2} - ESK^2 \frac{\partial^4 y_i}{\partial x^4}, \qquad \frac{\partial^2 z_i}{\partial t^2} = T_{0,i} \frac{\partial^2 z_i}{\partial x^2} - ESK^2 \frac{\partial^4 z_i}{\partial x^4} \tag{1}$$

where y_i and z_i are the transverse motions of the strings ($i = 1, 2$) in T and P directions at a position x and at a time t, μ is the mass per unit length, T_0 is the tension, E is the Young's modulus, S is the cross-sectional area and K is the radius of gyration. For a pinned string with length L, the n-th mode shape and the corresponding natural angular frequency are [3]:

$$\phi_n(x) = \sin(n\pi x/L), \qquad \omega_n = n2\pi f_0(1 + Bn^2)^{\frac{1}{2}} \tag{2}$$

where $f_0 = \left(T_{0,i}/\mu\right)^{\frac{1}{2}}/2L$ is the fundamental frequency in the absence of bending stiffness, and the inharmonic coefficient $B = \pi^2 ESK^2/T_{0,i}L^2$. String damping is later included as damping ratios in the state-space formulation; estimated through measurements, a constant damping ratio of $\zeta_T = \zeta_P = 5e^{-5}$ is adopted in this paper.

To represent the longitudinal (L) vibration of the strings, the equation of motion of a rod is adopted as:

$$\rho S \frac{\partial^2 u_i}{\partial t^2} = ES \frac{\partial^2 u_i}{\partial x^2} \tag{3}$$

where u_i is the longitudinal displacement of the strings ($i = 1, 2$). The natural frequencies for the longitudinal vibration are $\omega_{L,n} = 2\pi n(1/2L)\sqrt{E/\rho}$, where ρ is the density of the string. For the longitudinal motion, damping ratios ζ_L were obtained from literature [4, 5].

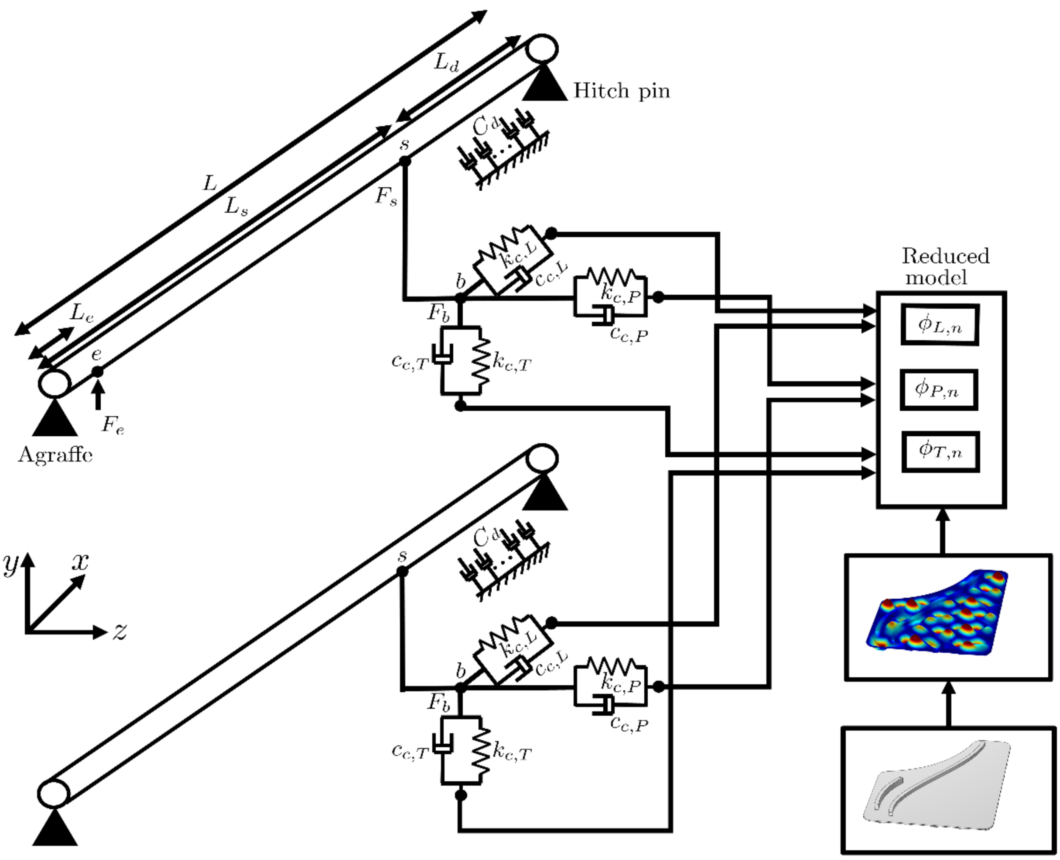

Figure 1. Schematic representation of two strings connected to the soundboard

2.2. Reduced modal soundboard

A finite element (FE) model is developed using COMSOL Multiphysics® of the soundboard of a grand piano. This is then used as a reference result to develop an equivalent and simpler modal representation of the soundboard at the bridge in three directions. The thickness of the soundboard is assumed to be constant; its edges are clamped, the bridges and the wooden stiffener beams - often referred to as ribs - are modelled as isotropic while the soundboard is given orthotropic material properties. The ribs are continuously connected to the soundboard by sharing nodes at their interfaces. The mechanical properties of the wood were obtained from literature [6] and correspond to Sitka spruce. The main parameters adopted in the FE model are shown in Table 1. A total of ~16800 tetrahedral solid elements were used.

Table 1. Dimensions and parameters for the FE model.

Description	Value	Description	Value
Soundboard width, m	1.37	**Poisson's ratio ν_{12}**	0.37
Soundboard length, m	1.0	**Poisson's ratio ν_{13}**	0.47
Young's modulus E_1, GPa	12.7	**Poisson's ratio ν_{23}**	0.43
Young's modulus E_2, GPa	1.04	**Shear modulus G_{12}, GPa**	1.0
Young's modulus E_3, GPa	0.48	**Shear modulus G_{13}, GPa**	0.96
Thickness, m	0.008	**Shear modulus G_{23}, GPa**	0.04
Density, $\mathrm{kg/m^3}$	488		

Although the FE model is based on a specific example, it is intended to represent a generic and realistic soundboard and it is used to evaluate the reference dynamic behaviour at the bridge in three directions, yielding the 3×3 mobility matrix at each frequency at the string-bridge connection points. However, the inclusion of a full dynamic model of the soundboard is not necessary and is computationally expensive. A simpler model can be sufficient [7, 8] to capture the main effects. In this work a reduced order modal model is adopted which was developed by retaining the dominant mode in each one-twelfth octave band and in each direction leading to three mode components (T, L, P) in each frequency band. The reducing procedure gives 102 modes out of a total of 800 found in the frequency range of interest, 20-7000 Hz. This approach is suitable since covers the whole frequency range and not only a number of modes located in a specific range.

The results are shown in Figure 2 for the driving point mobilities at the location at the soundboard bridge corresponding to the connection with a C2 string. Results are shown according to the full 3×3 driving point mobility matrix at the connection point:

$$\boldsymbol{Y} = \begin{bmatrix} \boldsymbol{Y}_{TT} & \boldsymbol{Y}_{TL} & \boldsymbol{Y}_{TP} \\ \boldsymbol{Y}_{LT} & \boldsymbol{Y}_{LL} & \boldsymbol{Y}_{LP} \\ \boldsymbol{Y}_{PT} & \boldsymbol{Y}_{PL} & \boldsymbol{Y}_{PP} \end{bmatrix} \tag{4}$$

where the subscripts indicate the responses and excitation directions, respectively. Results are presented omitting the symmetric terms of the mobility matrix. A constant damping ratio was used $\zeta_b = 0.03$ which is within the limits of what was encountered in the literature [6, 9, 10]. The mobilities obtained by the reduced modal model have a good agreement with the FE results, particularly at lower frequencies, where the soundboard resonances can interact with the string and therefore need to be represented adequately in the modal model. The main differences between the reduced and full model are in the PP and LP terms of the mobility, these deviations can be minimized by increasing the number of modes used in the modal reduction. This is not further explored in this work as more modes of the soundboard will yield in increased computational times.

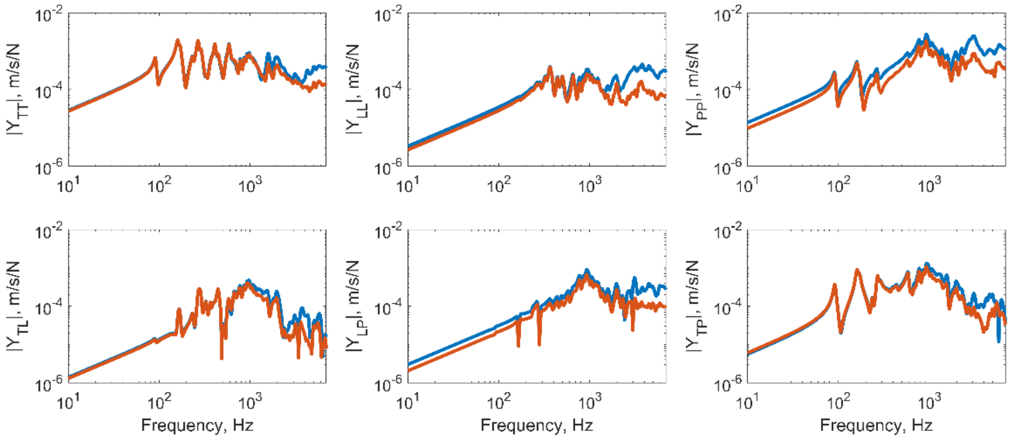

Figure 2. FE mobilities (blue) vs Reduced order model mobilities (red) at position C2

The resulting mode shapes are used in the state-space formulation described below, for coupling the string and the soundboard at the connection point b. The soundboard mode shapes are arranged in the matrix $\boldsymbol{\phi}_b$ given as

$$\boldsymbol{\phi}_b = \begin{bmatrix} \boldsymbol{\phi}_T \\ \boldsymbol{\phi}_L \\ \boldsymbol{\phi}_P \end{bmatrix}, \tag{5}$$

where each mode shape row vector $\boldsymbol{\phi}_T$, $\boldsymbol{\phi}_L$ and $\boldsymbol{\phi}_P$ contains 102 modes.

It is assumed that both unison strings share the same soundboard mobility at the bridge since they are within 1-2 cm of each other.

2.3. Connection between the string and the soundboard.

The connection between the string and the soundboard is modelled by means of a contact stiffness and damper [11, 12]. In this case the force in each direction is proportional to the relative displacement and velocity between string and soundboard in the corresponding direction as

$$\mathbf{F}_s = \begin{bmatrix} k_{c,T} & & \\ & k_{c,L} & \\ & & k_{c,P} \end{bmatrix} \begin{bmatrix} y_s - y_b \\ u_s - u_b \\ z_s - z_b \end{bmatrix} + \begin{bmatrix} c_{c,T} & & \\ & c_{c,L} & \\ & & c_{c,P} \end{bmatrix} \begin{bmatrix} \dot{y}_s - \dot{y}_b \\ \dot{u}_s - \dot{u}_b \\ \dot{z}_s - \dot{z}_b \end{bmatrix} \tag{6}$$

where k_c and c_c represent the stiffness and the damping of the contact zone respectively in the different directions. An expression for the contact stiffness $k_{c,T}$ is derived from Hertzian contact theory [13],

$$k_{c,T} = \frac{\pi L_c}{4} \frac{E_s E_w}{E_s(1-\nu_w^2) + E_w(1-\nu_s^2)} \tag{7}$$

where E_s, ν_s and E_w, ν_w are the Young's moduli and Poisson's ratios of the steel string and the wooden bridge and L_c is the length of the contact zone. The value of $k_{c,T}$ is in the order of $L_c E_w$ and for a small contact length $L_c \approx 0.01$ m is evaluated as 4.8×10^6 N/m; similar values are used for the contact stiffness in the other directions. The damping coefficients $c_{c,T}$, $c_{c,L}$ and $c_{c,P}$ are set at 10 Ns/m, and are used to provide numerical stability.

3. Numerical modelling

A time-domain model in state-space form is developed to describe the coupling between the different components. The hammer-string interaction is described first followed by the state-space formulation of the whole system.

3.1. Hammer excitation

The force exchanged between the hammer and one or more strings is described in terms of a non-linear power law [14], given by:

$$F_e = K_H \xi^p, \qquad \xi = \begin{cases} y_H - y_e & \text{if } y_H > y_e \\ 0 & \text{otherwise} \end{cases} \tag{8}$$

where ξ is the compression of the hammer and y_H and y_e are the displacement of the hammer and the string at the excitation point. The equation of motion of the hammer, represented as a mass, can be written as:

$$F_e = -m_H \ddot{y}_H, \tag{9}$$

where the parameters K_H, m_H and p correspond to the nonlinear stiffness, mass and power law coefficients obtained experimentally for piano hammers [15]. The term $\ddot{y}_H$ is the hammer acceleration.

3.2. State-space formulation

The system in state-space form can be expressed as [16]

$$\dot{\mathbf{x}} = \mathbf{A}\mathbf{x} + \mathbf{B}\mathbf{u} + \mathbf{B}_2\mathbf{F}_b \tag{10}$$

On the left-hand side of Eq.(10) the state vector $\mathbf{x} = (\mathbf{q}_1, \mathbf{q}_2, \mathbf{q}_b, \dot{\mathbf{q}}_1, \dot{\mathbf{q}}_2, \dot{\mathbf{q}}_b, y_H, \dot{y}_H,)^T$ contains the modal displacements $\mathbf{q}$ and velocities $\dot{\mathbf{q}}$ of the strings and the soundboard, the displacement of the hammer y_H, its velocity $\dot{y}_H$. The terms on the right-hand side are the state-space matrix (**A**) and two forcing terms related to the hammer-string interaction (**B**) and the string-bridge interaction ($\mathbf{B_2}$). These are described below.

The state-space matrix **A** can be written as:

$$\mathbf{A} = \left[\begin{array}{cccccc|cc}
\mathbf{0}_1 & & & \mathbf{I}_1 & & & & \\
& \mathbf{0}_2 & & & \mathbf{I}_2 & & & \\
& & \mathbf{0}_b & & & \mathbf{I}_b & & \\
-\mathbf{K}_1 & & & -\mathbf{C}_1 - \mathbf{C}_\mathrm{d} & & & & \\
& -\mathbf{K}_2 & & & -\mathbf{C}_2 - \mathbf{C}_\mathrm{d} & & & \\
& & -\mathbf{K}_b & & & -\mathbf{C}_b & & \\
\hline
& & & & & & 0 & 1 \\
& & & & & & 0 & 0
\end{array}\right] \tag{11}$$

where $\mathbf{K}$ and $\mathbf{C}$ are the diagonal modal stiffness and damping matrix of the two strings and soundboard (subscripts $1, 2$ and b) and have a size that depends on the number of modes adopted to describe each component. For the strings 100 modes are used considering two transverse motions and 5 for the longitudinal, leading to a total number of 205 modes for each string, while the reduced order model gave 102 modes for the soundboard as already described above. This provides enough modes in the frequency range of interest. The modal matrices of one string can be written as:

$$\mathbf{K}_1 = \begin{bmatrix} \omega_{T,1}^2 & & & & & & \\ & \omega_{L,1}^2 & & & & & \\ & & \omega_{P,1}^2 & & & & \\ & & & \ddots & & & \\ & & & & \omega_{T,n}^2 & & \\ & & & & & \omega_{L,n}^2 & \\ & & & & & & \omega_{P,n}^2 \end{bmatrix}$$
$$\mathbf{C}_1 = \begin{bmatrix} 2\zeta_{T,1}\omega_{T,1} & & & & & & \\ & 2\zeta_{L,1}\omega_{L,1} & & & & & \\ & & 2\zeta_{P,1}\omega_{P,1} & & & & \\ & & & \ddots & & & \\ & & & & 2\zeta_{T,n}\omega_{T,n} & & \\ & & & & & 2\zeta_{L,n}\omega_{L,n} & \\ & & & & & & 2\zeta_{P,n}\omega_{P,n} \end{bmatrix} \tag{12}$$

A distributed damper is applied to the duplex scaling segment of the strings to attenuate vibration that would propagate beyond the bridge. The distributed damping coefficient matrix that appears in Eq. (11) can be written as:

$$\mathbf{C_d} = \int_{L_s}^{L_s+L_d} c_d \boldsymbol{\Phi}_\mathbf{n}(x)\boldsymbol{\Phi}_\mathbf{n}^\mathbf{T}(x)dx, \tag{13}$$

where L_d is the length of segment between the bridge and the hitch pin and c_d is a damping coefficient that is chosen arbitrarily to mute the resonances associated with this segment but to avoid influencing substantially those mostly associated to the speaking length. In practice c_d represents the strip of felt woven into the strings at this location. Note that in most pianos the mid-low range tones have their duplex scaling segment muted by strips of felt while it is tuned and left free to vibrate in the higher register.

Assuming that the hammer strikes the strings in the transverse direction only and for the case in which the hammer strikes both strings, the modal forcing term **Bu** in Eq.(10) can be written as:

$$\mathbf{Bu} = \begin{bmatrix} \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} \\ \boldsymbol{\Phi}_{1,e}^T & \mathbf{0} \\ \mathbf{0} & \boldsymbol{\Phi}_{2,e}^T \\ \mathbf{0} & \mathbf{0} \\ - & - \\ 0 & 0 \\ -1/m_H & -1/m_H \end{bmatrix} \begin{bmatrix} F_{e,1} \\ F_{e,2} \end{bmatrix} \tag{14}$$

For the *una corda* case, **u** is just a scalar $u = F_{e,1}$ and matrix **B** is a column vector containing string mode shapes at the excitation point e of one string, as well as the inverse of the hammer mass:

$$\mathbf{B} = \begin{bmatrix} \mathbf{0} \\ \mathbf{0} \\ \mathbf{0} \\ \boldsymbol{\phi}_{1,e}^{T} \\ \mathbf{0} \\ \mathbf{0} \\ - \\ 0 \\ -1/m_H \end{bmatrix} \tag{15}$$

For a hammer excitation only provided in T direction, the mode shape vectors at the excitation point $\boldsymbol{\phi}_{1,e}$, $\boldsymbol{\phi}_{2,e}$ are written as:

$$\boldsymbol{\phi}_{1,e} = \boldsymbol{\phi}_{2,e} = [\phi_{T,1} \quad 0 \quad 0 \quad \phi_{T,2} \quad 0 \quad 0 \quad \dots \quad \phi_{T,n} \quad 0 \quad 0] \tag{16}$$

The remaining modal force term $\mathbf{B_2 F_b}$ couples the soundboard and the string and is given as:

$$\mathbf{B_2 F_b} = \begin{bmatrix} \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \boldsymbol{\phi}_{1,s}^{T} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \boldsymbol{\phi}_{2,s}^{T} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \boldsymbol{\phi}_{b}^{T} \\ - & - & - \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} -\mathbf{F}_{1,s} \\ -\mathbf{F}_{2,s} \\ \mathbf{F}_{1,s} + \mathbf{F}_{2,s} \end{bmatrix} \tag{17}$$

which introduces the modal contact force between the string and the soundboard. The mode shape matrix $\boldsymbol{\phi}_b$ is defined in Eq.(5) while the string mode shape matrices $\boldsymbol{\phi}_{1,s}$, $\boldsymbol{\phi}_{2,s}$ are written as:

$$\boldsymbol{\phi}_{1,s} = \boldsymbol{\phi}_{2,s} = \begin{bmatrix} \phi_{T,1} & 0 & 0 & \dots & \phi_{T,n} & 0 & 0 \\ 0 & \phi_{L,1} & 0 & \dots & 0 & \phi_{L,n} & 0 \\ 0 & 0 & \phi_{P,1} & \dots & 0 & 0 & \phi_{P,n} \end{bmatrix}_s \tag{18}$$

The multidirectional string-to-soundboard forces $\mathbf{F}_{1,s}$ and $\mathbf{F}_{2,s}$ are defined in Eq.(6). The physical displacements and velocities of the different parts of the system are calculated as

$$\mathbf{y} = \mathbf{Cx} \tag{19}$$

where the output vector $\mathbf{y}$ in Eq.(19) contains the physical velocities and displacements and the matrix $\mathbf{C}$ converts the modal coordinates to physical coordinates, and is written as:

$$\mathbf{C} = \begin{bmatrix} \boldsymbol{\Phi}_{1,s} & & & & & & & \\ & \boldsymbol{\Phi}_{2,s} & & & & & & \\ & & \boldsymbol{\Phi}_{b} & & & & & \\ & & & \boldsymbol{\Phi}_{1,s} & & & & \\ & & & & \boldsymbol{\Phi}_{2,s} & & & \\ & & & & & \boldsymbol{\Phi}_{b} & & \\ & & & & & & 1 & \\ & & & & & & & 1 \end{bmatrix} \tag{20}$$

The numerical time integration of Eq.(10) is performed in MATLAB using the 4th order Runge-Kutta method. An initial hammer impact velocity of 2.5 m/s was considered, and the resolution of the solution is determined by a sampling frequency of $f_s = 10 \times f_{max}$ where f_{max} is the highest expected (in)harmonic natural frequency of the string.

4. Results

The results of some example simulations are shown in terms of interaction forces between the hammer and the string(s) and between the strings and the soundboard. The latter can be used subsequently in a model of the vibration and sound radiation of the soundboard for sound synthesis.

4.1. Hammer-string contact force

When using the *una corda* pedal in a duplet string, the hammer will strike only one string, whereas in the case of normal playing the hammer will strike the two strings in the same manner. The different resulting contact forces for the two cases are shown in Figure 3. The hammer-string interaction is affected by using the *una corda* pedal; when hitting two strings the hammer imparts two equal forces in the two strings while the force is larger and of longer duration when the collision takes place with a single string.

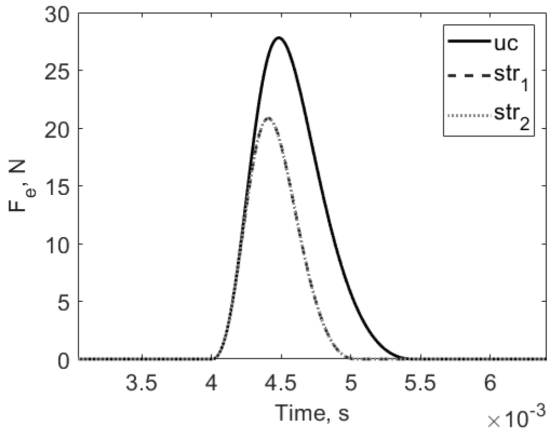

Figure 3. Hammer-string contact force. uc: *una corda*. str1: string one (nominal tension) in normal playing conditions. str2: string two (with tension modification) in normal playing conditions

4.2. Forces exerted by the strings on the soundboard

The forces exerted by each string on the soundboard are analysed in this section. The time histories of the forces and their corresponding spectra in each direction are presented. The resulting force acting on the soundboard can be computed as a summation of the string forces, per direction.

First the time histories are shown in Figure 4. The figures to the left show results for the two strings in the *una corda* case and the figures to the right show the corresponding results for the normal playing case. The three components of force are shown. For the *una corda* case, the second string (Figure 4(b)), not excited, initially increases its vibration level with time as power flows from the excited string into the soundboard and then back to the non-excited string. The transverse vibration component in both strings is significantly larger than the other components, mainly due to the hammer striking in this direction. Nonetheless, the excited string (Figure 4(a)), exhibits a longer decay (aftersound) of the transverse component, than in normal playing conditions (Figure 4(c, d)). This is caused by the coupling between the strings, where the increasing vibration of the passive string influences the excited string. In normal playing conditions (Figure 4(c, d)), the hammer strikes both strings, and the resultant forces transmitted to the soundboard are similar; the differences are caused by small differences in the tension of one of the strings.

The resultant forces exerted on the soundboard are a summation of the string components in each direction; these are shown in Figure 5. As expected, in normal playing conditions (Figure 5(b)) when the hammer strikes the two strings the overall transmitted force is larger than in the *una corda* case. However, as stated by Weinreich [1], in comparison to the attack the aftersound is relatively larger in the *una corda* case, where the vibration of the passive string increases with time.

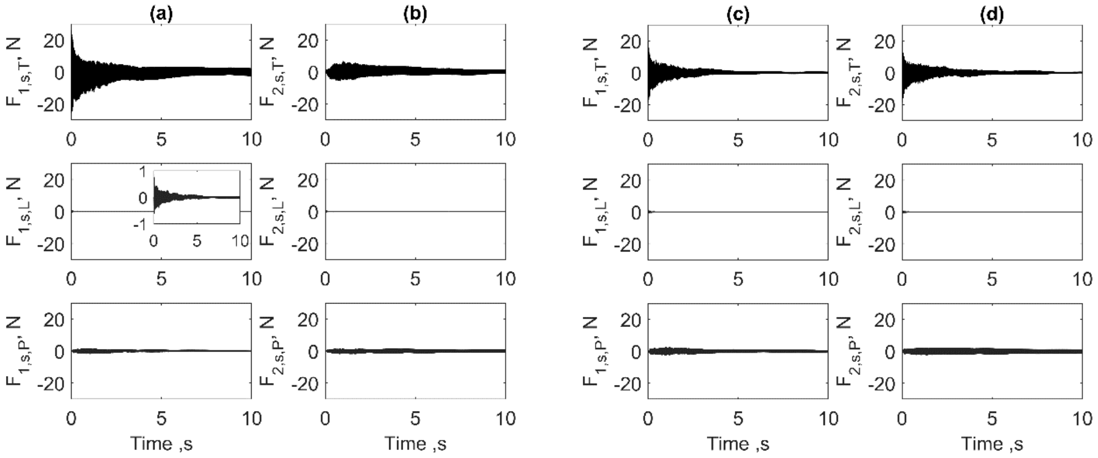

Figure 4. Forces exerted by the string on the soundboard. Left: *una corda.* Right*:* normal playing conditions. (a, c): string 1, (b, d): string 2.

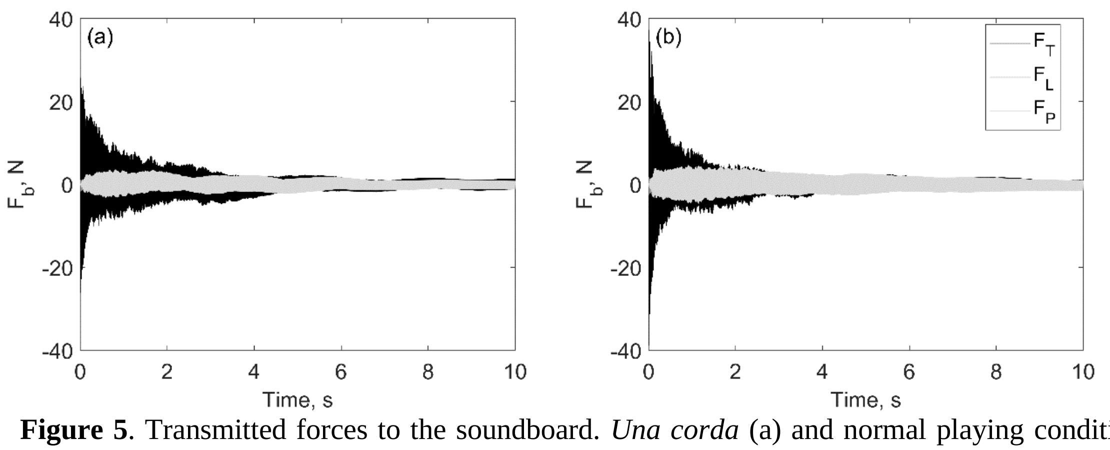

Figure 5. Transmitted forces to the soundboard. *Una corda* (a) and normal playing conditions (b).

It is of interest to note the time dependent tonal characteristics of the transverse component T of transmitted forces F_b. For this, spectrograms are shown in Figure 6, where a beating phenomenon can be observed for normal playing conditions, in which the level of the partials repeatedly decreases and then increases, whereas in the *una corda* case, the decay of the partials is more even.

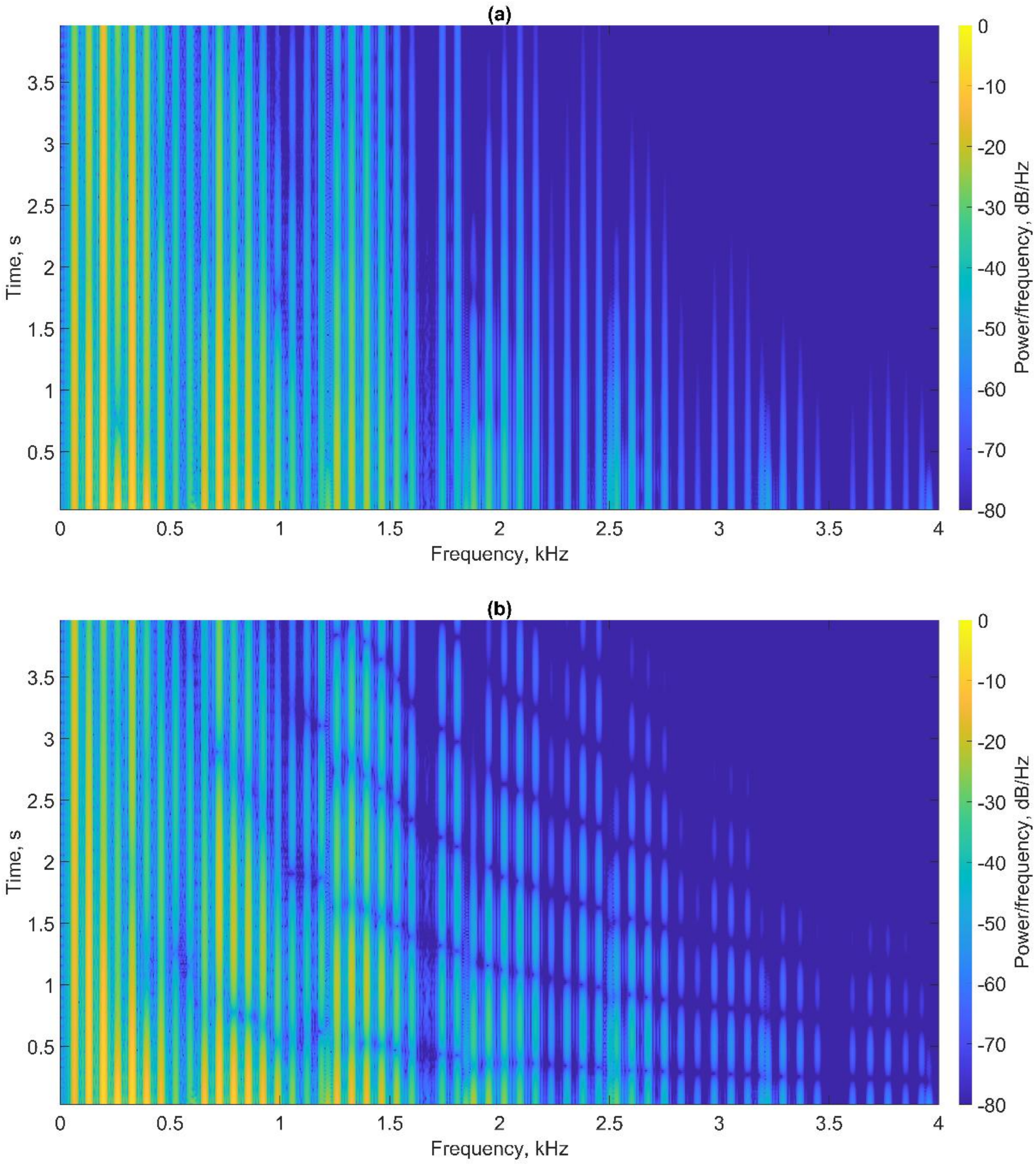

Figure 6. Spectrograms of the first 4 s and until 4 kHz. *Una corda* (a) and normal playing conditions (b).

5. Conclusions

The use of the *una corda* pedal can affect the vibration transmission from the string to the soundboard. The contact forces between hammer and strings are reduced in normal playing conditions due to the larger stiffness associated with striking more than one string, while the *una corda* playing will yield larger contact forces with increased duration. The forces transmitted to the soundboard by each string differ significantly. In normal playing conditions the aftersound is small relative to the attack produced by the hammer, while in the *una corda* case the excited string shows a larger aftersound, and the passive string's vibration initially increases with time. The resultant forces transmitted to the soundboard exhibit a beating phenomenon in normal playing conditions, while using the *una corda* pedal provides a more uniform and regular decay. The *una corda* pedal could be used for softer and slower piano passages where the decay or sustain of the sound is of importance.

The modelling performed in this work, showing the effects of sympathetic vibration in strings when using the *una corda* pedal, could be extended to include the sympathetic vibration of all the strings while using the sustain pedal. Such a model would have to include the transfer mobilities along the bridges and would be greatly benefited by the reduced soundboard model developed from the finite element model.

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