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Active control of acoustic scattering from a passively optimised spherical shell

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ABSTRACT

***At low frequencies, the sound power scattered from a spherical shell can be minimised by designing the material properties and thickness so that its mass and compressibility are the same as that of the displaced fluid. The scattered power is then dominated at higher frequencies by that due to the resonances of the structural modes of the shell, particularly the ovalling mode. The peaks in the scattered power due to structural resonances can be reduced somewhat by material damping but are more effectively attenuated with active control using structural actuators as secondary sources. Of particular interest are structural actuators and sensors that are distributed over the surface of the sphere, rather than just acting at single points. Simulations are presented of the scattered sound power of such a shell when subject to feedforward control, which assumes knowledge of both the incident and scattered acoustic sound fields, and structural feedback control, which only assumes that the velocity on the surface of the sphere can be measured*.**

1. INTRODUCTION

The scattering of sound is important in a number of applications, such as binaural sound reproduction, where the physical presence of the head plays an important role in the perceived sound [1], and in acoustic cloaking of objects [2], which is important in scenarios involving acoustic detection. The sound scattered from a body surrounded by a fluid can be calculated numerically, using finite elements or boundary elements for example, or analytically if the body has a simple shape, such as a sphere [3,4,5].

Bobrovnitskii [6,7] introduced an impedance-based approach to the analysis of sound scattering by assuming that the surface of the scattering body was divided into a large number of discrete elements, which are assumed to be small compared with a wavelength in the surrounding fluid. If the pressure and velocity over the surface are instead expressed in terms of a modal expansion, an entirely analogous analysis of scattering can be formulated. Assuming tonal excitation proportional to $e^{j\omega t}$, the vectors of complex total modal pressures and total modal velocities on the surface of the scattering body are denoted $\mathbf{p}_t$ and $\mathbf{v}_{t,}$ where $\mathbf{v}_t$ is measured normal and outward with respect to the surface. Each of these vectors is made up of contributions from the sound field incident on the scattering body, and contributions from the scattered sound field, so that $\mathbf{p}_t$ can be written as $\mathbf{p}_i$ plus $\mathbf{p}_s$ and $\mathbf{v}_t$ as $\mathbf{v}_i$ plus $\mathbf{v}_s$. Three input impedance matrices are then defined, which are the in-vacuo structural impedance matrix of the scattering body, $\mathbf{Z}_B$, the impedance matrix of the internal volume of

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the scattering body if filled with the surrounding fluid, $\mathbf{Z}_I$, and the outward radiation impedance matrix into the surrounding fluid, $\mathbf{Z}_R$, so that

$$\mathbf{p}_t = -\mathbf{Z}_B \cdot \mathbf{v}_t \quad (1) \qquad \mathbf{p}_i = -\mathbf{Z}_I \cdot \mathbf{v}_i, \quad (2) \qquad \mathbf{p}_s = \mathbf{Z}_R \cdot \mathbf{v}_s. \quad (3)$$

Using simple manipulations of the defining equations (1), (2) and (3), the vector of scattered surface pressures, $\mathbf{p}_s$, can be expressed in terms of the vector of incident surface pressures, $\mathbf{p}_i$, as [6,7]

$$\mathbf{p}_s = (\mathbf{Y}_R + \mathbf{Y}_B)^{-1} \cdot (\mathbf{Y}_I - \mathbf{Y}_B) \cdot \mathbf{p}_i = \mathbf{R} \cdot \mathbf{p}_i, \tag{4}$$

where the admittance matrices $\mathbf{Y}_B$, $\mathbf{Y}_R$ and $\mathbf{Y}_I$ are the inverses of the impedance matrices $\mathbf{Z}_B$, $\mathbf{Z}_R$ and $\mathbf{Z}_I$, assuming that these matrices are non-singular. Under the conditions of linearity and reciprocity, these matrices are also symmetric, and when all the processes involved are passive, the real parts of the matrices are positive definite and so all of their associated impulse responses are causal. It is important to note that despite being formulated in terms of the in-vacuo structural response of the body, the loading of the fluid on the structure, as well as the sound scattering, are all accounted for in equation (4).

These impedance matrices are fully populated in the original formulation using elemental radiators, but each could be diagonalised by choosing a model expansion involving either the structural modes of the body, for $\mathbf{Z}_B$, the interior acoustic modes of the space, for $\mathbf{Z}_I$, or the radiation modes, for $\mathbf{Z}_R$. The eigenvectors of any of the three impedance matrices could thus potentially be used to define this modal expansion. For the particular case of the scattering from a thin, uniform, empty spherical shell in an infinite fluid, however, an expansion in terms of spherical harmonics diagonalises all three impedance matrices. This expansion is truncated here to $n = N'$ terms, so that the pressure and velocity on the surface of the sphere are

$$p(\theta,\varphi) = \sum_{n=0}^{N} \sum_{m=-n}^{n} p(n,m)\, Y_n^m(\theta,\varphi)\,, \tag{5}$$
$$v(\theta,\varphi) = \sum_{n=0}^{N} \sum_{m=-n}^{n} v(n,m) Y_n^m(\theta,\varphi), \tag{6}$$

where $Y_n^m(\theta,\varphi)$ is the complex spherical harmonic of index (n,m) and $p(n,m)$ and $v(n,m)$ denote the modal amplitudes. The vectors of $N = (N' + 1)^2$ modal pressures and velocities are then defined as

$$\mathbf{p} = [p(0,0),\ p(1,-1),\ p(1,0),\ p(1,1) \dots\ p(N,-N) \dots\ p(N,0) \dots\ p(N,N)]^T, \tag{7}$$
$$\mathbf{v} = [v(0,0),\ v(1,-1),\ v(1,0),\ v(1,1) \dots\ v(N,-N) \dots\ v(N,0) \dots\ v(N,N)]^T. \tag{8}$$

In the case of a uniform spherical shell, all three impedance matrices are only dependant on the index n, and the diagonal elements associated with the n-th terms of the two acoustic impedance matrices can then be written as [8]

$$Z_I(n) = j\rho c \frac{j_n(ka)}{j_n'(ka)}\,, \tag{9}$$

$$Z_R(n) = -j\rho c \frac{h_n(ka)}{h_n'(ka)}\,, \tag{10}$$

where ρ and c are the density and speed of sound in the surrounding fluid, $j_n(ka)$ and $h_n(ka)$ are n-th order spherical Bessel function of the first kind and spherical Hankel function of the second kind and the prime superscript denotes derivation in respect to the normalized frequency, ka, where k is the acoustic wavenumber in the surrounding fluid and a is the radius of the sphere. The modal impedance of the spherical shell, is given by [4]

$$Z_B(n) = j\frac{\rho_s c_p}{\Omega}\frac{h}{a}\frac{(\Omega^2 - \Omega_{n1}^2)(\Omega^2 - \Omega_{n2}^2)}{\Omega^2 - (1+\beta^2)(v + \lambda_n - 1)}\ , \tag{11}$$

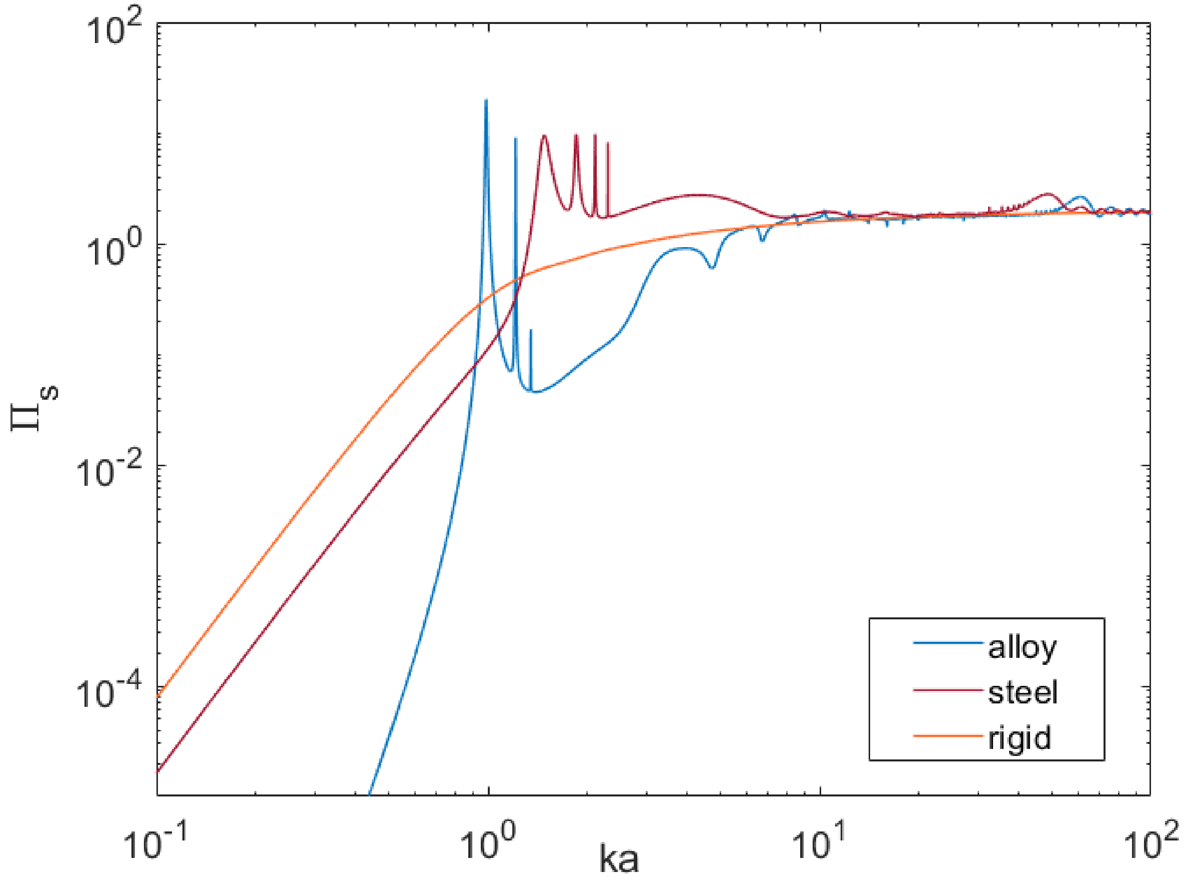


Figure 1: The normalised scattered power, in water, for spherical shells made of steel and of alloy, as described in the text, compared with that of a rigid sphere.

where ρ_s is the density of the shell, $c_p^2 = E_s/[\rho_s(1-v^2)]$ with E_s and v being the Young`s modulus and Poisson`s ratio of the shell, h is the shell thickness, a is the shell radius, β^2 is $h^2/(12a^2)$, λ_n is $n(n+1)$, and Ω is $\omega a/c_p$ with $\omega = kc$, which can also be written as $\Omega = (c/c_p)ka$. The values of Ω_{n1} and Ω_{n2} are the in-vacuo natural frequencies of vibration of the shell and are given by the solutions to the equation

$$\begin{gathered}\Omega^4 - [1 + 3v + \lambda_n - \beta^2(1 - v - \lambda_n^2 - v\lambda_n)]\Omega^2 + \\ +(\lambda_n - 2)(1 - v^2) + \beta^2[\lambda_n^3 - 4\lambda_n^2 + \lambda_n(5 - v^2) - 2(1 - v^2)] = 0\ ,\end{gathered} \tag{12}$$

It is convenient to write the impedance of the shell in normalised form as

$$Z_B(n) = \rho c \zeta_n(ka), \tag{13}$$

where $\zeta_B(n)$ is the in-vacuo impedance of the shell normalized by the characteristic acoustic impedance of the fluid, ρc.

The diagonal elements of the modal form of equation (4) then leads directly to an expression for the ratio between the n-th spherical harmonic component of the scattered pressure and that of the incident pressure [15,5]

$$\frac{\boldsymbol{p_s}(n)}{\boldsymbol{p_i}(n)} = \frac{Y_I(n) - Y_B(n)}{Y_R(n) + Y_B(n)} = -\frac{h_n(ka)}{j_n(ka)} \frac{j_n(ka) + j\zeta_B(n) j_n'(ka)}{h_n(ka) + j\zeta_B(n) h_n'(ka)} . \tag{14}$$

Using this formulation, the normalised sound power of a spherical shell in response to an incident plane-wave, which is proportional to the spatial integral of the mean square scattered pressure in the far-field, can be calculated as [5]

$$\Pi_s = \frac{W_s}{W_i} = \frac{4}{(ka)^2} \sum_{n=0}^{N} (2n+1) \left| \frac{j_n(ka) + j\zeta_n(ka) j_n'(ka)}{h_n(ka) + j\zeta_n(ka) h_n'(ka)} \right|^2 , \tag{15}$$

Figure 1 shows the normalised scattered power in water for two spherical shells of different materials and thicknesses, as a function of the normalised frequency ka, together with that for a rigid sphere. At low frequencies the normalised power scattered by the steel shell, $\rho_s = 7{,}700$ kg/m^3, $E_s = 190$ GPa, $\nu = 0.28$, $h/a = 2.3\%$, is proportional to $(ka)^4$, which is the same proportionality as the rigid shell. The properties of the "alloy" shell, $\rho_s = 10{,}000$ kg/m^3, $E_s = 72.9$GPa, $\nu = 0.28$, $h/a = 3.33\%$, have been chosen so that its low-frequency stiffness and mass match that of a sphere of water, so that the low frequency scattering due to the $n = 0$ and $n = 1$ spherical harmonics is suppressed [10,11]. The normalised scattered power is then proportional to $(ka)^8$ at low frequencies and hence much smaller than a rigid sphere. Although the alloy shell is modelled here as a thin, uniform, empty shell with appropriate mechanical properties, it might be realised in practice by a metal shell of a suitable thickness, surrounded by a soft coating with a similar density to the fluid, but with a thickness chosen to give the required overall stiffness [12]. In both cases shown in Figure 1 the scattered power at higher frequencies, from around ka=1 to 3, is dominated by peaks due to the structural resonances of the fluid-loaded shell, particularly corresponding to the n=2, ovalling, n=3 and n=4 vibration modes.

2. ACTIVE FEEDFORWARD CONTROL

A frequency domain feedforward control formulation can be used to calculate the optimal performance of an array of secondary forces in minimising the scattered power, assuming knowledge of the incident and scattered fields. This allows evaluation of the best possible performance with a given number of secondary sources, without having to be concerned with the sensing of the reference or of the error signals, or with the implementation of a practical controller, and so can be used as the first step in a hierarchical design approach for active control [9].

The shell is assumed to be controlled with L internal forces distributed over a spherical cap of angle θ_0, each of which has a magnitude f_l acting at (θ_l, φ_l) that generates a modal pressure of

$$p_l(n,m) = -\left[\frac{Z_B(n)}{Z_B(n) + Z_R(n)}\right] \frac{f_l}{a^2} \left[\frac{\mathrm{P}_{n-1}(\cos\theta_0) - \mathrm{P}_{n+1}(\cos\theta_0)}{(2n+1)(1-\cos\theta_0)}\right] \overline{Y}_n^m(\theta_l, \varphi_l) , \tag{16}$$

where the overbar denotes complex conjugation. The scattered modal pressure after control with L secondary point-forces is thus

$$p_{sc}(n,m) = p_s(n,m) - \left[\frac{Z_B(n)}{Z_B(n) + Z_R(n)}\right] \sum_{l=1}^{L} \frac{f_l}{a^2} \left[\frac{\mathrm{P}_{n-1}(\cos\theta_0) - \mathrm{P}_{n+1}(\cos\theta_0)}{(2n+1)(1-\cos\theta_0)}\right] \overline{Y}_n^m(\theta_l, \varphi_l) , \tag{17}$$

which can be written in vector form as

$$\mathbf{p}_{sc} = \mathbf{p}_s - \mathbf{B} \cdot \tilde{\mathbf{p}}_c , \tag{18}$$

where $\tilde{\mathbf{p}}_c = \frac{1}{a^2}[\,f_1,\ \ f_2,\ \ f_3 \dots\ \ f_L]^T$ is the vector of L control forces, which has a tilde to denote these are discrete rather than modal pressures, and $\mathbf{B}$ is equal to $\mathbf{Z}_B[\boldsymbol{Z}_{\boldsymbol{B}} + \boldsymbol{Z}_{\boldsymbol{R}}]^{-1}$ times the N by L matrix $\mathbf{S}$, where the l-th column of $\mathbf{S}$ has elements as $\left[\frac{\mathrm{P}_{n-1}(\cos\theta_0) - \mathrm{P}_{n+1}(\cos\theta_0)}{(2n+1)(1-\cos\theta_0)}\right]\bar{Y}_n^m(\theta_l, \varphi_l)$.

The scattered power after control is given by

$$W_{sc} = \frac{a^2}{2}\mathrm{Re}\{\mathbf{p}_{sc}^{\mathrm{H}} \cdot \mathbf{v}_{sc}\} = \frac{a^2}{2}\mathbf{p}_{sc}^{\mathrm{H}} \cdot \mathrm{Re}\{\mathbf{Y}_R\} \cdot \mathbf{p}_{sc}\,, \tag{19}$$

since $\mathbf{v}_{sc}$ is given by $\mathbf{Y}_R \cdot \mathbf{p}_{sc}$, which is a quadratic function of $\tilde{\mathbf{p}}_c$. The scattered pressure can thus be minimised by the vector of control forces given by [9]

$$\tilde{\mathbf{p}}_c^{(\mathrm{opt})} = [\mathbf{B}^{\mathrm{H}} \cdot \mathrm{Re}\{\mathbf{Y}_R\} \cdot \mathbf{B}\,]^{-1} \cdot \mathbf{B}^{\mathrm{H}} \cdot \mathrm{Re}\{\mathbf{Y}_R\} \cdot \mathbf{R} \cdot \mathbf{p}_i\,, \tag{20}$$

where equation (4) has been used to relate $\mathbf{p}_s$ to $\mathbf{p}_i$. Figure 2 shows, on an enlarged scale, the results of minimising the scattered sound power of the alloy shell using either one distributed secondary force, with $\theta_0 = \pi/10$, facing the incident wave, or two distributed secondary forces, facing towards and away from the incident wave. Significant control of the scattered sound power can be achieved at the peaks due to the structural resonances.

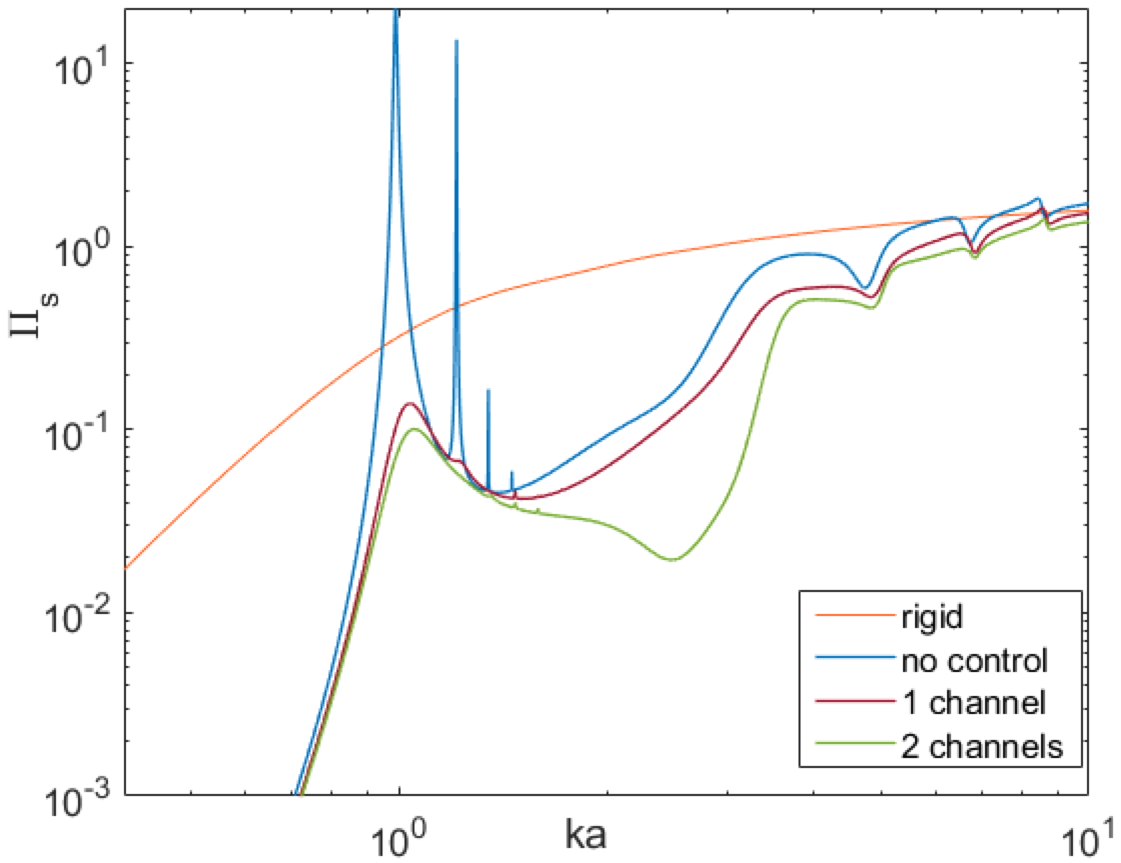


Figure 2: Optimal results of using active feedforward control with one or two distributed force actuators to minimise the scattering from the alloy shell in water.

3. ACTIVE FEEDBACK CONTROL

The effect of active feedback control on scattering can be calculated by initially considering the consequences of feedback control on the in-vacuo response of the shell. Figure 3(a) shows the physical arrangement in which the signals from K discrete velocity sensors is fed to L internal point force actuators via a feedback controller matrix, **H**. The effect of such a controller on the modal velocity in response to a general modal pressure excitation is shown in Figure 3(b).

In the absence of control and using equation (1), the vector of modal velocities in response to a general vector of modal pressures is

$$\mathbf{v} = -\mathbf{Y}_B \cdot \mathbf{p}\,, \tag{21}$$

In the presence of active control with an array of internal point-force sources, as in Section 2, the modal pressures acting on the shell are modified, although in this in-vacuo case there is no fluid loading so that the matrix **B** in equation (18) reduces to **S**, so that

$$\mathbf{v} = -\mathbf{Y}_B \cdot (\mathbf{p} - \mathbf{S} \cdot \tilde{\mathbf{p}}_c), \tag{22}$$

For the feedback arrangement shown in Figure 3(a), the secondary forces are due to feedback from the velocities at the K sensor positions, $\tilde{\mathbf{v}}_c$, which also has a tilde to denote discrete rather than a modal velocities, via the feedback controller **H**, so that

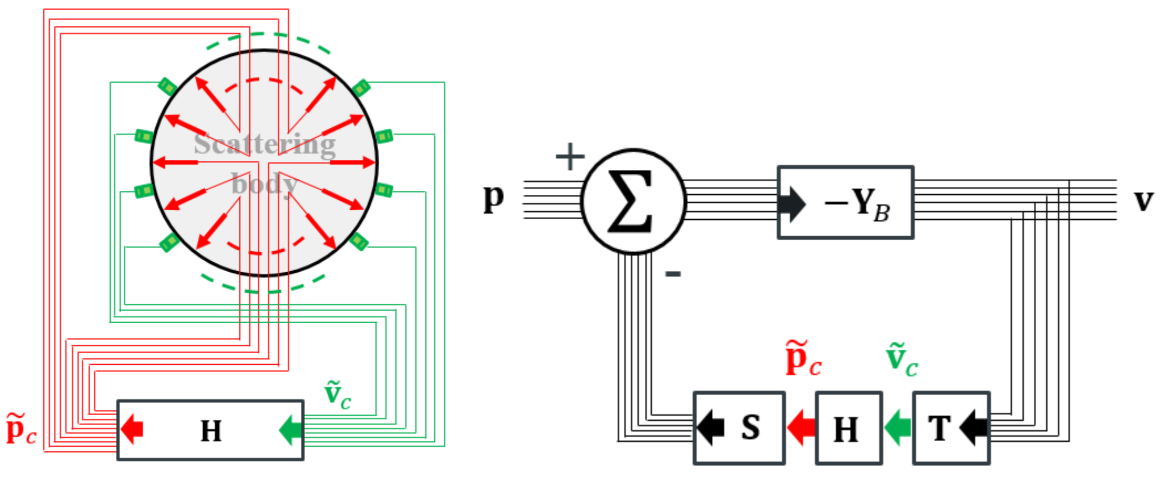
$$\tilde{\mathbf{p}}_c = \mathbf{H} \cdot \tilde{\mathbf{v}}_c, \quad (23) \qquad \text{where} \qquad \tilde{\mathbf{v}}_c = \mathbf{T} \cdot \mathbf{v}\,, \quad (24)$$

$\mathbf{v}$ being the vector of modal velocities, and **T** is a K by N transformation matrix, which, from equation (6), has elements of the form $Y_n^m(\theta_k, \varphi_k)$. Thus,

$$\mathbf{v} = -\mathbf{Y}_B(\,\mathbf{p} - \mathbf{S} \cdot \mathbf{H} \cdot \mathbf{T} \cdot \mathbf{v})\,, \tag{25}$$

and so,

$$\mathbf{v} = -[\mathbf{I} + \mathbf{Y}_B \cdot \mathbf{S} \cdot \mathbf{H} \cdot \mathbf{T}]^{-1} \cdot \mathbf{Y}_B \cdot \mathbf{p}\,. \tag{26}$$



(a) (b)

Figure 3: Feedback control using secondary point-force actuators driven by the measured velocities at discrete sensors on the surface of the body (a), and the block diagram of the equivalent modal feedback system (b).

The modal pressures can this be expressed as

$$\mathbf{p} = -\mathbf{Z}_B \cdot [\mathbf{I} + \mathbf{Y}_B \cdot \mathbf{S} \cdot \mathbf{H} \cdot \mathbf{T}] \cdot \mathbf{v} = -\mathbf{Z}_B^{(cl)} \cdot \mathbf{v}, \tag{27}$$

where $\mathbf{Z}_B^{(cl)}$ is the overall modal impedance for the shell with closed-loop feedback control, which can be written as

$$\mathbf{Z}_B^{(cl)} = \mathbf{Z}_B + \mathbf{S} \cdot \mathbf{H} \cdot \mathbf{T}\,. \tag{28}$$

Although $\mathbf{Z}_B$ is diagonal for a spherical harmonic expansion, the matrix $\mathbf{S} \cdot \mathbf{H} \cdot \mathbf{T}$ is generally not diagonal. Nevertheless, the matrix $\left[\mathbf{Z}_B^{(cl)}\right]^{-1}$ can now be used instead of $\mathbf{Y}_B$ in equation (4) to give the vector of scattered modal pressures after feedback control in terms of the vector of incident modal pressures, and, hence, the scattered power after control can be calculated using equation (19).

In the absence of an incident field, the matrix of "plant" responses, $\mathbf{G}_c$, between the point-force actuators and the discrete velocity sensors is

$$\tilde{\mathbf{v}}_c = \mathbf{T} \cdot [\mathbf{Z}_B + \mathbf{Z}_R]^{-1} \cdot \mathbf{S} \cdot \tilde{\mathbf{p}}_c = \mathbf{G}_c \cdot \tilde{\mathbf{p}}_c\ , \tag{29}$$

since the force actuators have to overcome both the impedance of the shell and the radiation impedance. If there are the same number of actuators as sensors and they are collocated, then $\mathbf{S}$ is equal to $\mathbf{T}^{\mathrm{H}}$ so

$$\mathbf{G}_c = \mathbf{T} \cdot [\mathbf{Z}_B + \mathbf{Z}_R]^{-1} \cdot \mathbf{T}^{\mathbf{H}}\,, \tag{30}$$

which is entirely passive. The stability of the closed-loop system is thus guaranteed provided the feedback gain matrix, $\mathbf{H}$, is also passive [13,14, 16] and is unconditionally stable for direct velocity feedback.

Several designs of feedback controller are possible. The simplest is decentralised local velocity feedback, for which $\mathbf{H}$ is equal to $\gamma \boldsymbol{I}$, where γ is the gain of each local feedback loop. Figure 4 shows the effect of decentralised velocity feedback on the scattered power from the alloy shell in Figure 1 with one or two distributed force actuators, as above, and collocated distributed velocity sensors. The feedback gain, γ, is chosen to give a reasonable compromise between supressing the original lightly damped structural resonances and not exciting higher-order resonances by pinning the structure [13, 14]. This feedback controller is able to effectively supress the very lightly damped resonances between about ka equal to 1 and 3.

In the limiting case, where it is assumed that there are as many actuators and sensors as there are modes, $L = K = N$, and assuming that the transducers are collocated and positioned so as to actuate and sense all of the modes such that $\mathbf{T}^{\mathrm{H}} \cdot \mathbf{T}$ is a good approximation to the identity matrix, then perfect control of all the modes would be possible. In this case, the closed-loop impedance of the shell could, in principal, be set equal to the input impedance of the volume of the scattering body filled with fluid, $\mathbf{Z}_I$, so that the scattering would be completely suppressed. Setting equation (28) equal to $\mathbf{Z}_I$ in this case leads to the feedback controller would having the form [6,7]

$$\mathbf{H} = \mathbf{T} \cdot (\mathbf{Z}_I - \mathbf{Z}_B) \cdot \mathbf{T}^{\mathrm{H}} \tag{31}$$

The stability of this feedback controller is by no means guaranteed, however, and was found not to be stable in the cases considered here.

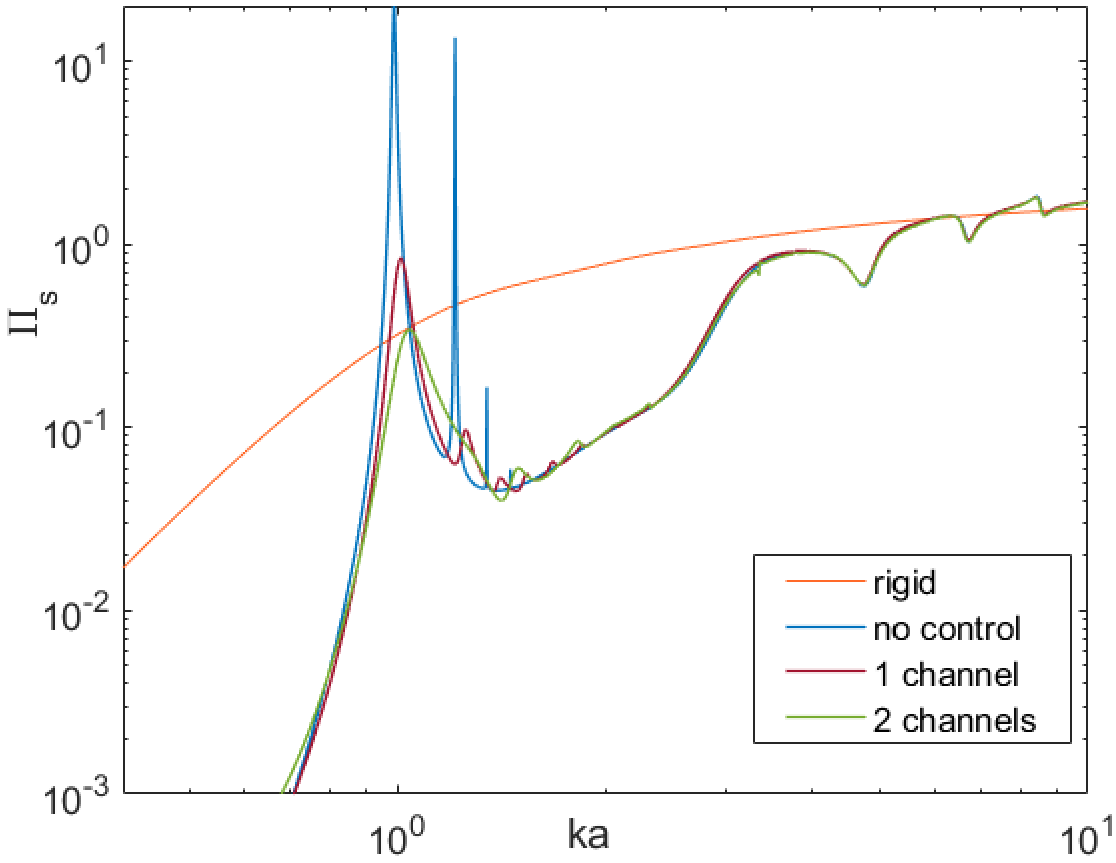


Figure 4: Normalised scattered sound power from the alloy shell after active feedback control with one or two distributed force actuators and collocated distributed velocity sensors using decentralised velocity feedback.

4. CONCLUSIONS

A modal formulation for acoustic scattering from a flexible body is reviewed. This general theory is shown to take a particularly simple form in the case of scattering from a thin, uniform, empty spherical shell surrounded by an infinite fluid, since the internal and external acoustic modes, and the structural modes, are then all spherical harmonics. Examples of scattering are calculated for spherical shells of different materials, and it is shown that the scattering due to both the $n = 0$ and the $n = 1$ spherical harmonics can be suppressed in a "alloy" shell with a suitable choice of the shell's material properties.

This formulation is then used to calculate the effect of active feedforward or feedback control on the scattering, using distributed forces acting on the flexible spherical shell as secondary actuators. The effect of both forms of active control in reducing the scattering from the composite shell is mainly to due to the suppression of a few lightly damped structural resonances.

5. ACKNOWLEDGEMENTS

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