Object Allocation Over a Network of Objects: Mobile Agents with Strict Preferences

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Abstract

In recent work, Gourvès, Lesca, and Wilczynski propose a variant of the classic housing markets model where the matching between agents and objects evolves through Pareto-improving swaps between pairs of adjacent agents in a social network. To explore the swap dynamics of their model, they pose several basic questions concerning the set of reachable matchings

In this work, we pursue the same set of questions under a natural variant of their model. In our model, the social network is replaced by a network of objects, and a swap is allowed to take place between two agents if it is Pareto-improving and the associated objects are adjacent in the network.

Problem Setting

- Let A denote a set of agents and let B denote a set of objects such that $|A| = |B|$.
- Each agent is intially matched to a distinct object.
- Objects are connected to each other by a graph $G = (B, E)$.
- Each agent has preferences over all of the objects.
- If agent a is matched to object b, agent a' is matched to object b', $b' \succ_a b$, $b \succ_{a'} b'$, and (b, b') belongs to E, then a and a' are allowed to swap. After the swap, a is matched to b' and a' is matched to b.

Motivation

Consider a cloud computing environment with a large number of servers (objects) connected by a network that are available to rent. A set of customers (agents) are each interested in renting one server. The servers vary in CPU capacity, storage capacity, physical security, and rental cost. Varying customer workloads and requirements result in varying customer preferences over the servers. Rather than attempting to globally optimize the entire matching of customers to servers, it might be preferable to allow local swaps between adjacent servers to gradually optimize the matching. Given that customer workloads are likely to vary significantly over time, an optimization strategy based on frequent local updates might outperform a strategy based on less frequent global updates. Alternatively, one can envision a system that performs occasional global updates to optimize the matching, and that relies on local updates to maintain a reasonable matching between successive global updates.

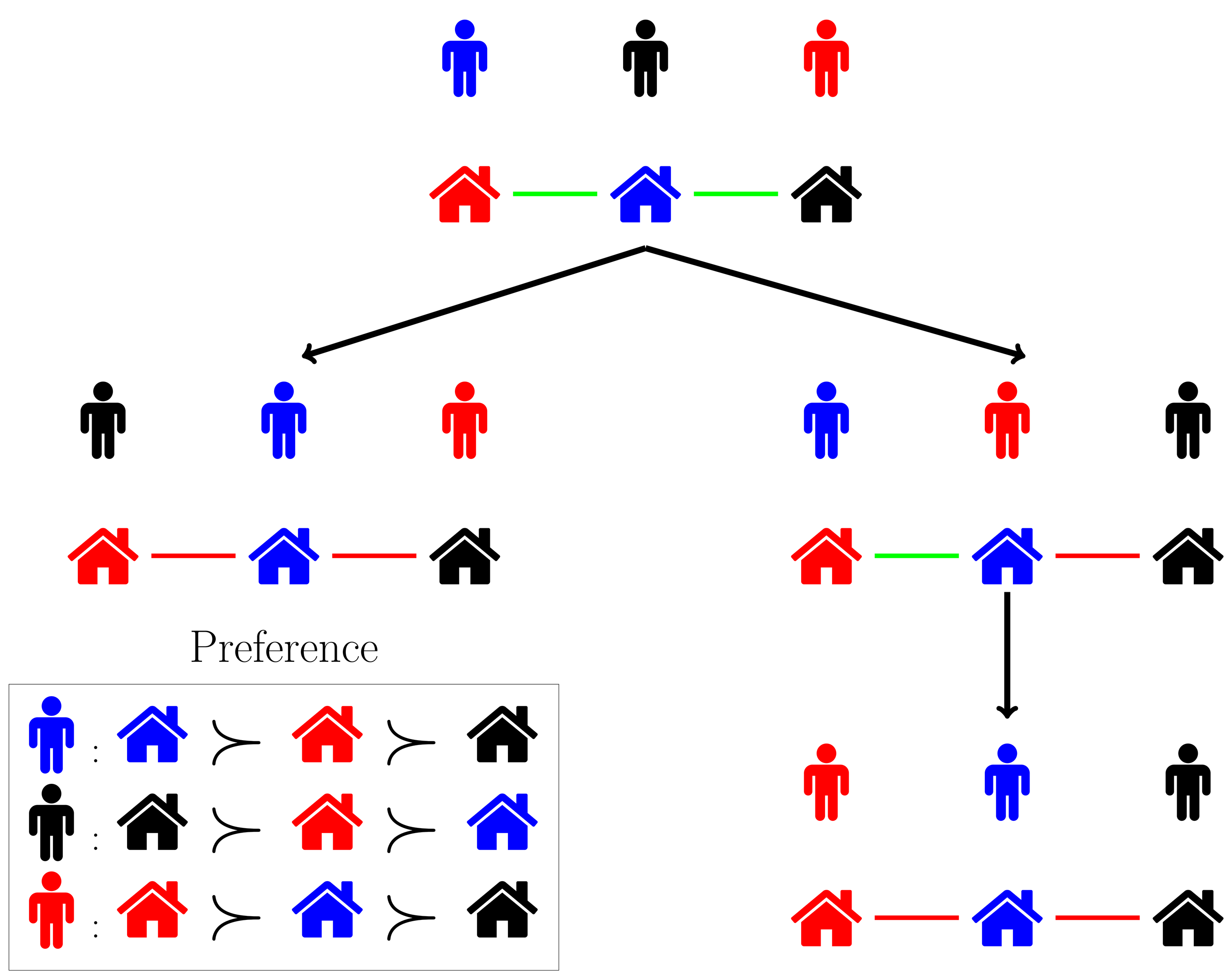
Reachablity Problems

We study the following three reachability problems.

- Reachable Object: Is there a sequence of swaps that results in agent a being matched to object b?
- Reachable Matching: Is there a sequence of swaps that results in a specified target matching?
- Pareto Efficiency: Find a reachable matching that is not Pareto-dominated by any other reachable matching.

Example

Consider the following instance with three objects arranged in a path, and three agents. The initial matching is shown at the top of the diagram. The arrows indicate valid swaps.



Consider the following two examples of reachability problems for the above instance (which would both be answered postively): is there a sequence of swaps that results in the red agent being matched to the red object, and is there a sequence of swaps that results in each agent being matched to the object of the same color? Moreover, note that the matching on the bottom right is the Pareto-efficient matching.

Results

	Reachable Object	Reachable Matching	Pareto Efficiency
Star	poly-time	(poly-time)	poly-time
Path	poly-time	(poly-time)	poly-time
Generalized Star	NP-complete	(poly-time)	poly-time
Tree	(NP-complete)	poly-time	open
Clique	NP-complete	NP-complete	NP-hard

Table: This table presents our results for various quesions. The results in parentheses follow directly from other table entries.

We present an $O(n^2)$ time algorithm for Reachable Object on paths in the agent-moving model, which is much faster than the known $O(n^4)$-time algorithms for Reachable Object on paths in the object-moving model. (Here n denotes the number of agents/objects; the size of the input is quadratic in n since the preference list of each agent is of length n.)

Open Problems

- We present a polynomial-time algorithm for Pareto Efficiency on generalized stars in the agent-moving model. This problem remains open in the object-moving model of Gourvès et al.
- It would also be interesting to resolve Pareto Efficiency on trees in either the agent-moving or object-moving model.

References

[1] Laurent Gourvès, Julien Lesca, and Anaëlle Wilczynski. Object allocation via swaps along a social network. In *Proceedings of the 26th International Joint Conference on Artificial Intelligence*, pages 213–219, 2017.

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