Optimal Crowdfunding Design  

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Mechanism Design Problem

Standard Scheme: One Price-Threshold Pair

- Each consumer \( i \) has private value \( v_i \in [v_L, v_H] \)
- Each consumer chooses action:
  - \( a_i = 1: \) contributes to the product
  - \( a_i = 0: \) does not contribute

Bayesian Nash Equilibrium (BNE):

\[
E_{v_i \sim p}[u_i(a_i(v_i), a_{-i}(v_{-i}))] \geq E_{v_i \sim p}[u_i(a_i(v_i), a_{-i}(v_{-i}))]
\]

\( \forall i, v_i, a_i \)

Non-trivial equilibrium:

\[
E_{v_i \sim p} \left[ \sum_{i=1}^{n} u_i(v_i) \right] \geq N
\]

Seller: maximize expected profit by choosing optimal pair \( (N, \tau) \)

Results:

- Unique symmetric BNE
- Maximized expected profit:
  \[
  R^* = \max \{ R(1, v_H, 1), R(1, v_L) \}
  \]

where

\[
R(1, v_H) = (A - \gamma B)(1 - p)^n + \sum_{k=1}^{n}(A - \gamma \max[B - k(v_H - \tau_0), 0] + \max[k(v_H - \tau_0) - \beta, 0]) \left( 1 - \frac{1}{n} \right) (1 - p)^{n-k} p^k
\]

\[
R(1, v_L) = A - \gamma \max[B - n(v_L - \tau_0), 0] + \max[n(\tau_L - v_L) - \beta, 0]
\]

\( A: \) future profit \; \( \gamma: \) interest rate for outside option \; \( B: \) initial cost to start production \; \( \tau_L, \tau_H: \) marginal cost for each product

Crowdfunding Campaign

When the number of consumers who commit to pre-buy exceeds the threshold, the crowdfunding succeeds and the seller gets the pre-buy payments.

Variation 1: Bulk Discount

Two Price-Threshold Pair

- Additional threshold \( N_2 \) \( (1 \leq N \leq N_2) \) for the discounted price \( \tau_2 \) \( (\tau_2 \leq \tau) \)
- Each consumer’s utility:

\[
u_i(a_i, a_{-i}) = \begin{cases} 
  v_i - \tau, & \text{if } a_i = 1, N \leq \sum_{j=1}^{n} a_j < N_2 \\
  v_i - \tau_2, & \text{if } a_i = 1, \sum_{j=1}^{n} a_j \geq N_2 \\
  0, & \text{otherwise}
\end{cases}
\]

- Seller: maximize expected profit by choosing optimal pair \( (N, \tau, N_2, \tau_2) \)

Results:

- Unique symmetric BNE
- Depending on the relationship among \( v_L, v_H, \tau, \tau_2 \)
- Maximized expected profit:

\[
R^* = \max \{ R(1, v_H, 1, v_H), R(1, v_L, v_L) \}
\]

where

\[
R(1, v_H, 1, v_H) = R(1, v_L, 1, 1) = R(1, v_L)
\]

- Setting an additional pair of threshold and price does NOT help increase the seller’s expected profit!

In reality:

- No exact number of consumers, but a rough estimation
- Relatively large \( \tau \) and small \( N \) to guarantee a successful crowdfunding and some amount of money
- Discounted price \( (\tau_2) \) with a larger threshold \( (N_2) \) for possibly large amount of money

Variation 2: Product Differentiation

Two Threshold

- Additional threshold \( N_1 \) \( (1 \leq N_1 \leq N) \) for simplified version
- Each consumer’s value \( v_i \in [v_L, v_H] \) for simplified version

\( - v_i < v_L < v_H \) also follows binary distribution \( p \)

- Each consumer’s utility:

\[
u_i(a_i, a_{-i}) = \begin{cases} 
  v_i - \tau, & \text{if } a_i = 1, N_1 \leq \sum_{j=1}^{n} a_j < N \\
  v_i - \tau_1, & \text{if } a_i = 1, \sum_{j=1}^{n} a_j \geq N_1 \\
  0, & \text{otherwise}
\end{cases}
\]

- Seller: maximize expected profit by choosing optimal pair \( (N_1, N, \tau) \)

Results:

- Unique symmetric BNE
- Depending on the relationship among \( v_L, v_H, \tau \)
- Lemma: for any integer \( \bar{N} \) satisfying \( N \leq \bar{N} \leq n + 1, \)

\[
\arg \max_{N_1 \leq \bar{N} \leq N} R(N_1, N, \tau) = N_1 \text{ or } \bar{N}
\]

- To maximize the profit, the seller should ONLY provide the standard version, or ONLY provide the simplified version!
- Additional threshold is NOT the cause for possible increase in profit!

In reality:

- Simplified version requires less initial and marginal cost.
- Also considered to guarantee a successful crowdfunding and some amount of money

Raising funding for developing and producing new products