Rank Aggregation by Dissatisfaction Minimisation in the Unavailable Candidate Model

Arnaud Grivet Sébert, Nicolas Maudet, Patrice Perny and Paolo Viappiani
LIP6, UMR 7606 Sorbonne Université, Paris, France

Introduction

Several approaches have recently been proposed to address the problem of candidates' unavailability in social choice, among which the *unavailable candidate model* proposed in [1] where the optimal rankings are computed by minimisation of the expected number of disagreements over all the possible subsets of available candidates. Lu and Boutilier [1] provide a clear justification for producing a ranking instead of a single winner: the output ranking serves as a very compact decision policy to select the best available candidate as ``winner''.

- C: set of all the candidates
- $top_r(S)$: best ranked candidate in subset S, for ranking (or voter) r
- $p \in [0,1]$: probability for any candidate to be unavailable, leading to a Bernoulli distribution over the subsets of C
- The optimal rankings minimise the sum of disatisfaction among all the voters, where the disatisfaction between r and r' is

$$D_p(r,r') = \sum_{S \subseteq C} p^{m-|S|}(1-p)^{|S|}\mathbf{1}[top_r(S) \neq top_{r'}(S)]$$

Generalized unavailable candidate model

- Binary disagreement in [1] -> a voter is satisfied if her preferred available candidate is elected and fully unsatisfied otherwise.
- In our work, the voter's satisfaction varies more smoothly and depends on the rank she gives to the winner.
- Two opposed ways to measure the satisfaction of the voters.

In the following,

- ρ measures the disutility associated with a rank
- P: the distribution of probability on the sets of available candidates

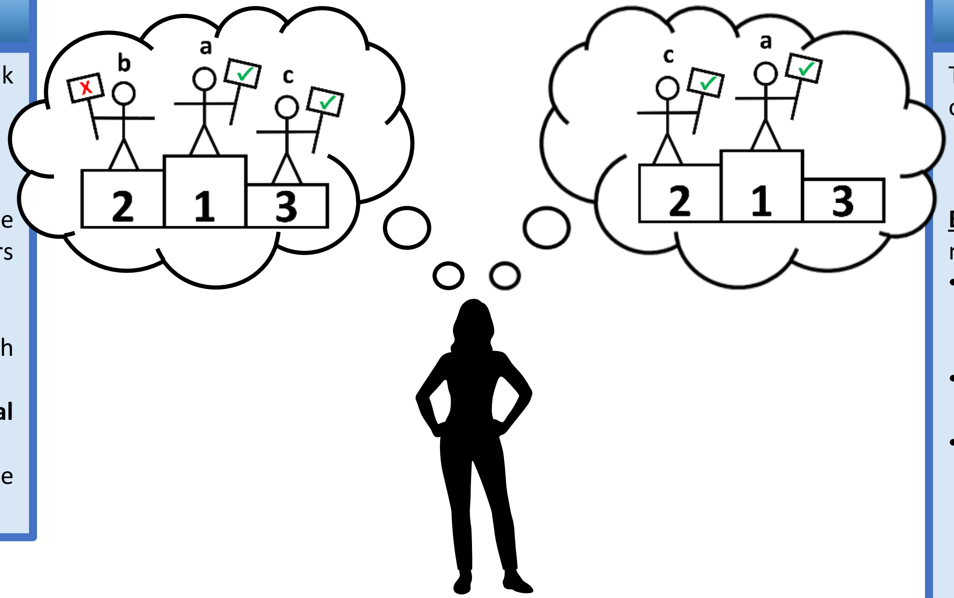
Ex-ante approach

The disutility felt by the voter v is computed considering the rank of the candidates among all the candidates of C:

$$\hat{\Delta}_{\rho,P}(v,r) \coloneqq \boldsymbol{E}_{S\sim P}[\rho\left(v\left(top_r(S)\right)\right) - \rho\left(v\left(top_v(S)\right)\right)]$$

<u>Ex. 1</u>: Let C={a,b,c}, ρ=(0,1,2), P the uniform distribution. We consider 4 voters with the preference order (a,b,c) and 7 voters with the order (c,a,b). The only optimal ranking is **(a,c,b)**.

- The ex-ante dissatisfaction rule amounts to a **scoring rule** with score –ρ
  - ➢ finding an optimal ranking can be done in **polynomial time**
  - ➢ the optimal rankings do not depend on the distribution P



Ex-post approach

The disutility felt by the voter v is computed considering the rank of the candidates *within* the set of available candidates:

$$\Delta_{\rho,P}(v,r) \coloneqq \boldsymbol{E}_{S\sim P}[\rho\left(v_S\left(top_r(S)\right)\right) - \rho\left(v_S\left(top_v(S)\right)\right)]$$

<u>Ex. 2</u>: With the same setting as for Ex. 1, the only optimal ranking is now **(c,a,b)**.

- Under reasonable assumptions on P and ρ, any optimal ranking is also a **Kemeny consensus**
  - ➢ the ex-post dissatisfaction rule **NP-hard**.
- We exhibit a **polynomial-time approximation scheme**
  - ➢ the ex-post rule can be handled in practise.
- Outline of the algorithm:
  - ❖ While there is a dominant candidate (satisfying a property ensuring it is first in any optimal ranking) append it to the output ranking and remove it from C
  - ❖ Choose and order K candidates in C such that their contribution to $\Delta_{\rho,P}(v,r)$ is minimal
  - ❖ Randomly order the remaining candidates.

Voting rules covered by our model

Scoring rules, positional Spearman distance.	Kemeny rule, rules that minimise the expected number of disagreements under different probability distributions ([1], [2], [3]).

[1] Tyler Lu and Craig E Boutilier. 2010. The unavailable candidate model: a decision-theoretic view of social choice. InProceedings of the 11th ACM conference on Electronic commerce. 263–274.
[2] Katherine A Baldiga and Jerry R Green. 2013. Assent-maximizing social choice. Social Choice and Welfare 40, 2 (2013), 439–460.
[3] Hugo Gilbert, Tom Portoleau, and Olivier Spanjaard. 2020. Beyond PairwiseComparisons in Social Choice: A Setwise Kemeny Aggregation Problem. InThirty-Fourth AAAI Conference on Artificial Intelligence. 1982–1989.