Learning Competitive Equilibria in Noisy Combinatorial Markets

Enrique Areyan Viqueira, Cyrus Cousins & Amy Greenwald

Why do we care?
We want markets to run efficiently even when buyers can’t perfectly evaluate bundles. This is a common situation in practice.

For example, in spectrum auctions, telecommunication companies bid for spectrum licenses.

A company’s bid reflects future expected profit and thus depends on unobservable factors: future customer demand, future competitors, etc.

Experimental Results
The goal of our experiments is to robustly evaluate the empirical performance of our algorithms. We evaluate our algorithms on unit-demand valuations and the local synergy value model.

Unit-demand valuations. A unit-demand buyer has value for single-items packages. We constructed four different distributions over unit-demand valuations, each capturing distinct qualitative features of markets with unit-demand buyers (for details, please read our paper).

Figure 1 summarizes our unit-demand results. The plots show the number of buyers ($y$-axis) and the number of goods ($x$-axis) for each distribution. Elicitation with pruning consistently requires fewer confidence interval reports from buyers than the baseline, and darker colors indicate higher savings.

Elicitation Algorithms
Baseline: estimate all values via some concentration inequality (e.g., Hoeffding’s inequality).

Elicitation with Pruning.
Initially, all buyer, bundle pairs are active. Ask every buyer for rough value estimates (say up to “large” $\varepsilon_0 > 0$) for all bundles.

Repeat some number of times, $t = 0, 1, \ldots T$:
• Optimally allocate goods to buyers per current value estimates. Let $V_t$ be this allocation’s value.
• For each active buyer, bundle pair:
  (i) Suppose the buyer gets the bundle.
  (ii) Optimally allocate all other goods to buyers. – If $V_t$ is higher than value accrued by (i) + (ii), then deactivate buyer, bundle pair.

• Increase value estimates accuracy: $\varepsilon_{t+1} \leftarrow \varepsilon_t / 2$

We proved that pruning criteria (i) + (ii), preserves the expected market’s competitive equilibrium.

Expectation is over all active buyers. We present a market model with noisy buyer valuations and two learning algorithms that provably learn the market’s equilibria in our model.

References
MODEL values, only confidence intervals around them.

When buyers don’t communicate their exact values, only confidence intervals around them.

For example, Anna gets $\bullet$ and $\bigstar$ at 10 (basketball fan).

Bob gets $\bigstar$ at 100 (soccer fan).

Instead, Anna and Bob can provide confidence intervals around their values.

But, buyers might not know their values exactly.

Instead, Ana and Bob can provide confidence intervals around their values.

For example, with 95% probability:

Bob values $\bigstar$ at $100 \pm 10$ (soccer fan – mostly).

Goal: estimate a market’s competitive equilibrium when buyers don’t communicate their exact values, only confidence intervals around them.

In particular, our methodology is flexible enough to work in any combinatorial market. Minimal modifications will allow us to inject domain-dependent knowledge, an exciting future research direction.

In a Nutshell
In a combinatorial market, buyers have preferences over bundles or packages of goods.

Computing the value of a bundle is often complicated, for example, the value of showing ads to users, the value of oil drilling sites, etc.

We present a market model with noisy buyer valuations and two learning algorithms that provably learn the market’s equilibria in our model.

Model
A set of goods, for example: $\bigstar$, $\bullet$, $\bigstar$, $\bigstar$.

A set of buyers, let’s call them Ana and Bob. Buyers have values for packages of goods.

For example, Anna values $\bullet$ and $\bigstar$ at 10 (basketball fan).

Bob values $\bigstar$ at 100 (soccer fan).

In our example, there are $2^4 = 8$ many packages.

An allocation is a partition of goods for buyers. For example, Anna gets $\bigstar$ and $\bullet$, Bob gets $\bullet$.

A competitive equilibrium is an allocation and prices that make all market participants happy: the seller maximizes its revenue and the buyers maximize their utility (value minus payment) over all possible allocations at the fixed prices.

But, buyers might not know their values exactly.

Instead, Ana and Bob can provide confidence intervals around their values.

For example, with 95% probability:

Bob values $\bigstar$ at $100 \pm 10$ (soccer fan – mostly).

Goal: estimate a market’s competitive equilibrium when buyers don’t communicate their exact values, only confidence intervals around them.

Future Research
Our learning algorithm shows promise since it can learn markets well with far fewer reports from users compared to a baseline. Still, there is significant room for improvement.

Table 1: local synergy value model results. Elicitation with pruning is often more data efficient, achieves smaller errors, and results in outcomes where buyers are (almost) happy.

Table 1: local synergy value model results. Elicitation with pruning is often more data efficient, achieves smaller errors, and results in outcomes where buyers are (almost) happy.

Contact Information
Lab Web: glamor.cs.brown.edu
Personal Web: www.enriqueareyan.com
Contact Email: eareyan@brown.edu

Figure 1: sample efficiency of elicitation with pruning on unit-demand markets results.

Local synergy value model (Scheffel et.al. (2012)). Telecommunication service providers might value different bundles of radio spectrum licenses differently, depending on whether the licenses in the bundle complement one another. For example, a bundle including New Jersey and Connecticut might not be very valuable unless it also contains New York City.

The local synergy value model is a simple model of regional complementarities. For various target errors $\varepsilon$, Table 1’s left column reports confidence interval report savings, the middle column reports the actual error achieved, and the right column a metric of buyers happiness (zero is maximal happiness).