Maximizing Influence-Based Group Shapley Centrality

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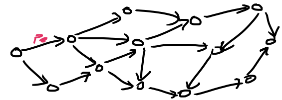
Motivation

"expected number of nodes reached from S"

Influence Maximization

Given: network $G=(V,E,p)$, $k\in\mathbb{N}$
Find: $S\subseteq V$, $|S|\leq k$ maximizing $\sigma(S)$



Classical Result: $(1-1/e-\varepsilon)$-approx. via Greedy Algorithm

Question: What if there are co-existing seeds $T\subseteq V\setminus S$?

> T known : trivial
> T unknown: nothing can be done!?

i.e. T follows probability distribution inspired by cooperative game theory

⟶ Assume T to be known probabilistically

Influence-Based Group Shapley Centrality

"expected increment S gives to random T"

$$\Phi^{sh}(S) := \mathbb{E}_{T\sim p^S}\left[\sigma(T\cup S)-\sigma(T)\right]$$

Shapley Value distribution:

> random permutation π of $V\setminus S\cup\{v_S\}$:

π: | T | v_S | |

Formally:

$$p^S(T)=\frac{|T|!\,(n-|S|-|T|)!}{(n-|S|+1)!}$$

⟶ Generalizes [CT17] from single nodes to k-sets.

[CT17] Chen and Teng. Interplay between Social Influence and Network Centrality. A Comparative Study on Shapley Centrality and Single Node Influence. WWW 2017.

The Max-Shapley-Group Problem

Max-Shapley-Group

Given: network $G=(V,E,p)$, $k\in\mathbb{N}$
Find: $S\subseteq V$, $|S|\leq k$ maximizing $\Phi^{sh}(S)$

RR-sets, see Borgs et al (SODA 14), Tang et al (SIGMOD 14)

Lemma [IGS via RR]:

$$\Phi^{sh}(S)=n\cdot\mathbb{E}_R\left[\frac{\mathbb{1}_{R\cap S\neq\emptyset}}{|R\setminus S|+1}\right]$$

$\xrightarrow[\text{bounds}]{\text{Concentration}}$ Max-Shapley-Group = "Harmonic-Max-Hitting-Set"

Results

- **Hardness of Approximation**

α-approximation for Max-Shapley-Group $\Longrightarrow$ $\alpha/8$-approximation for Densest-k-Subgraph

> GAP-ETH: no $1/n^{o(1)}$-approximation
> Unique Games with small set expansion: no constant approximation

⟶ Unlikely to find better than n^{-c}-approximation!

- **Approximation Algorithm**

greedily maximizing approx. of $n\cdot\mathbb{E}_R\left[\frac{\mathbb{1}_{R\cap S\neq\emptyset}}{|R|}\right]$ yields $\frac{(1-1/e)}{k}-\varepsilon$-approximation