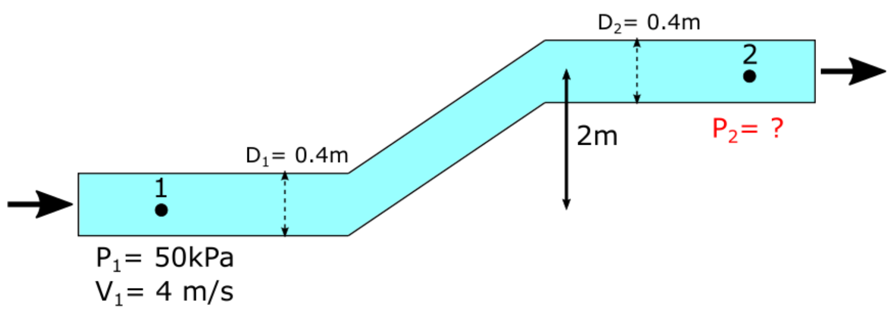
Worked Example Problems: Bernoulli's Equation

$$\frac{P_1}{\rho g} + z_1 + \frac{V_1^2}{2g} = \frac{P_2}{\rho g} + z_2 + \frac{V_2^2}{2g}$$

The objective in all three of the following worked example problems is to determine the pressure at location 2, P_2. For all three problems the gravitational constant, g, can be assumed to be $9.81 m/s^2$ and the density of water, ρ, as $1000 kg/m^3$. All pipes can be assumed to have circular cross-sections at all points.

Question 1



Solution

The first point to notice in this problem is the constant diameter of the pipe. If the diameter is the same at both locations, the area is the same at both locations. If the area is unchanged, based on flow continuity ($Q = VA$), the velocity is the same at both locations. Therefore the velocity head terms on either side of the equation cancel out.

$$\frac{P_1}{\rho g} + z_1 = \frac{P_2}{\rho g} + z_2$$

We will place our height reference line at the the level of lower of the two points, location 1. The static head is a measurement of vertical distance from this line. Therefore, as location 1 is on the line, $z_1 = 0$. The diagram indicates location 2 is 2m heigher than this. Therefore, $z_2 = 2$.

$$\frac{P_1}{\rho g} = \frac{P_2}{\rho g} + 2$$

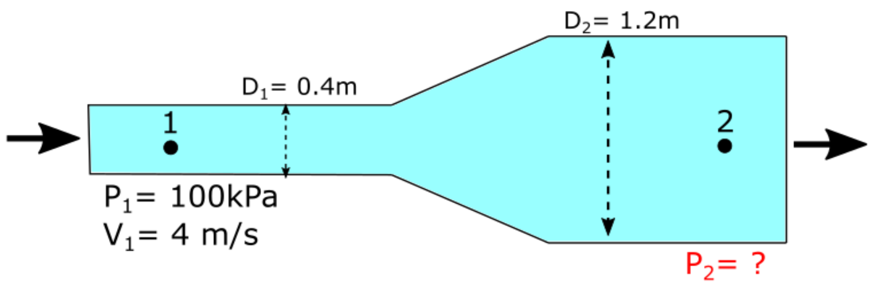
We now input our values for the gravitational constant and water density, provided at the start of the page. We can also input our value of P_1, provided in the question.

$$\frac{50 \times 10^3}{1000 \times 9.81} = \frac{P_2}{1000 \times 9.81} + 2$$

Rearrange for the one unknown.

$$P_2 = 30.4kPa$$

Question 2



Solution

Although the pipe expands, the centreline remains at the same height. As both the locations 1 and 2 are at the same height, the static head terms on either side of Bernoulli's equation cancel out. This could also be thought of as placing the height reference line along the two locations.

$$\frac{P_1}{\rho g} + \frac{V_1^2}{2g} = \frac{P_2}{\rho g} + \frac{V_2^2}{2g}$$

Using the diameter at location one and the area/diameter relationship for a circle, $A = \frac{\pi d^2}{4}$, the area of the pipe for location one can be calculated as $A_1 = 0.126m^2$.

The flow can then be calculated, using $Q = VA$, as $Q = 0.5m^3/s$.

Using the same area rule, the area of the pipe at location 2 can be obtained as, $A_2 = 1.13m^2$. Flow continuity states flow must remain constant along the pipe, meaning $Q_1 = Q_2$. V_2 can then be calculated by simply dividing, $\frac{Q_2}{A_2} = V_2 = 0.44m/s$.

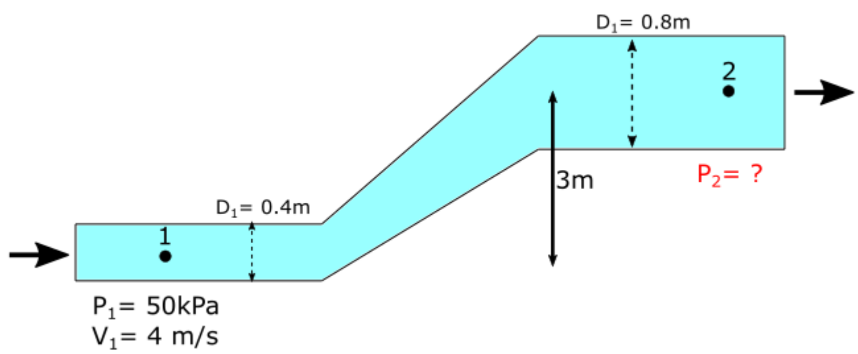
The values of the pressure at location one, the velocities at both locations,the gravitational constants and the density of water can all be put into the equation.

$$\frac{100\times10^3}{1000\times9.81}+\frac{4^2}{2\times9.81}=\frac{P_2}{1000\times9.81}+\frac{0.44^2}{2\times9.81}$$

Solve for the one unknown.

$$P_2 = 107.9kPa$$

Question 3



Solution

Our height reference line will be placed at the lower of the two locations, meaning $z_1 = 0$ and $z_2 = 3$.

$$\frac{P_1}{\rho g}+\frac{V_1^2}{2g}=\frac{P_2}{\rho g}+3+\frac{V_2^2}{2g}$$

The velocity value at location 2 can be deduced from flow continuity in the same manner as the previous question, $V_2 = 1m/s$.

All known values are in putted into the Bernoulli equation.

$$\frac{50\times10^3}{1000\times9.81}+\frac{4^2}{2\times9.81}=\frac{P_2}{1000\times9.81}+3+\frac{1^2}{2\times9.81}$$

Solve for the one unknown, P_2.

$$P_2 = 28.1kPa$$