Inclusive semi-leptonic decay rates of heavy mesons from lattice QCD

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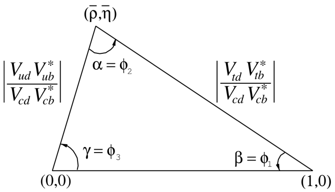
Flavour physics and the CKM matrix

- Quark flavour physics is an active area of research focusing on indirect search for signal of physics beyond the Standard Model (SM)
- Quark flavours mixing is parametrised by the elements of the Cabibbo-Kobayashi-Maskawa (CKM) matrix
- CKM matrix elements determined matching calculations with measurements
- Precise determination of CKM matrix elements is challenging

$$\begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix}$$



[PDG, CKM review]

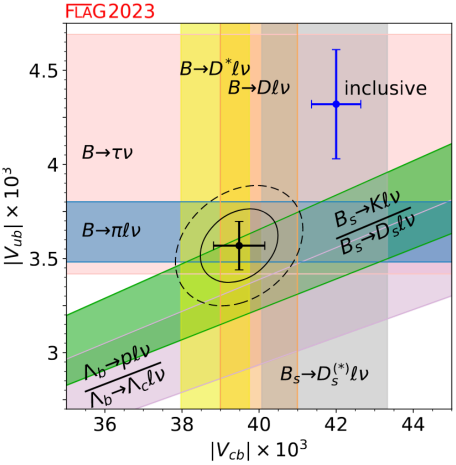
A long standing tension . . .

There is a perisitent tension between the **inclusive** and **exclusive** determination of the CKM matrix elements $|V_{cb}|$ and $|V_{ub}|$:

	$\|V_{cb}\|$	
Inclusive	$(42.00 \pm 0.64) \cdot 10^{-3}$	OPE
Exclusive	$(39.48 \pm 0.67) \cdot 10^{-3}$	LQCD

	$\|V_{ub}\|$	
Inclusive	$(4.32 \pm 0.29) \cdot 10^{-3}$	OPE
Exclusive	$(3.57 \pm 0.13) \cdot 10^{-3}$	LQCD

[FLAG '23]



$|V_{cb}|$ puzzle

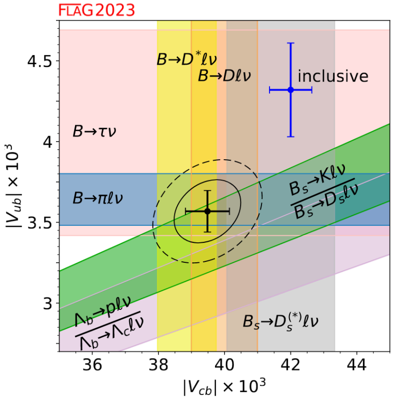
The PDG avarage of $|V_{cb}|$ is: $(40.8 \pm 1.4) \cdot 10^{-3}$

Even though it is unlikely to signal new physics, understanding the $|V_{cb}|$ puzzle is important because:

i. signal something not yet understood in exclusive or inclusive analysis with possible implications affecting $R(D^*)$

ii. limited accuracy of $|V_{cb}|$ affects FCNC studies in an important way

[Gambino, Jung, Shacht, *Phy. Lett. B*, 1905.08209]



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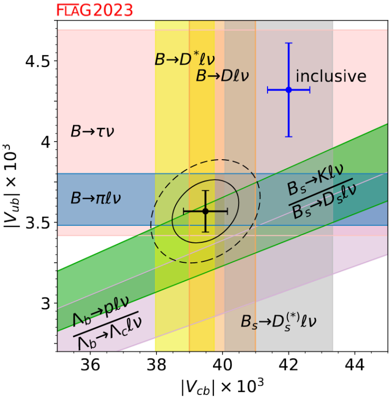
➜ **How can we address this issue?**

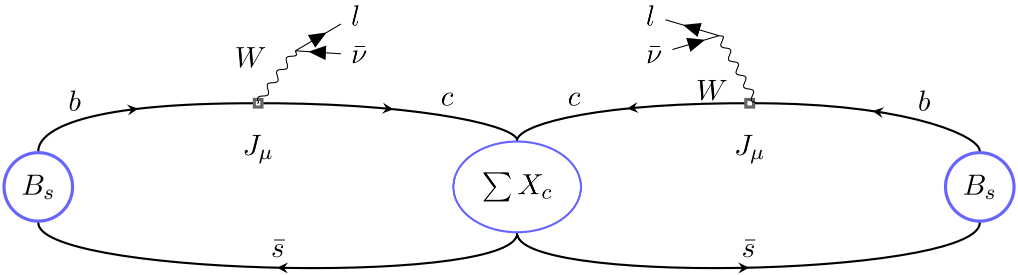


Inclusive semileptonic B-decays

$$\frac{d\Gamma}{dq^2 dq^0 dE_\ell} = \frac{G_F^2 |V_{cb}|^2}{(2\pi)^3} L_{\mu\nu} W^{\mu\nu}$$

$$W_{\mu\nu}(\omega, \boldsymbol{q}) = \frac{(2\pi)^3}{2m_{B_s}} \langle B_s(\mathbf{0}) | J_\mu^\dagger(0) \delta(\hat{H} - \omega) \delta^3(\hat{\boldsymbol{P}} + \boldsymbol{q}) J_\nu | B_s(\mathbf{0}) \rangle$$

$\Rightarrow$ encapsulate the **non-perturbative** QCD contribution



Calculation strategy [Gambino, Hashimoto, **PRL**, 2005.13730]

After performing E_ℓ integration, we work in the B_s-meson rest frame and write in compact form:

$$\frac{24\pi^3}{G_F^2|V_{cb}|^2}\frac{d\Gamma}{d\boldsymbol{q}^2} = \sum_{l=0}^{2} |\boldsymbol{q}|^{2-l} \int_{\omega_{min}}^{\omega_{max}} d\omega \; X(\omega, \boldsymbol{q}^2)$$

with

$$\omega_{min} = \sqrt{m_{D_s}^2 + \boldsymbol{q}^2}, \quad \omega_{max} = m_{B_s} - |\boldsymbol{q}|$$

where $X(\omega, \boldsymbol{q}^2)$ is a linear combinations of $W^{(l)}$:

$$X(\omega, \boldsymbol{q}^2) = \sum_{l=0}^{2} (\omega_{max} - \omega)^l W^{(l)}(\omega, \boldsymbol{q}^2)$$

$$W^{(0)} = W^{00} + \sum_{i,j=1}^{3} \frac{q^i}{\sqrt{\boldsymbol{q}^2}} \frac{q^j}{\sqrt{\boldsymbol{q}^2}} W^{ij} + \frac{q^i}{\sqrt{\boldsymbol{q}^2}} (W^{0i} + W^{i0})$$

$W_{\mu\nu}$ vanishes for $\omega < \omega_{min}$, so we introduce the kernel

$$\Theta^{(l)}(\omega_{max} - \omega) = (\omega_{max} - \omega)^l \theta(\omega_{max} - \omega)$$

so that:

$$Z^{(l)}(\boldsymbol{q}^2) = \int_0^\infty d\omega \; \Theta^{(l)}(\omega_{max} - \omega) \; W^{(l)}(\omega, \boldsymbol{q}^2)$$

and

$$\frac{24\pi^3}{G_F^2 |V_{cb}|^2} \frac{d\Gamma}{d\boldsymbol{q}^2} = \sum_{l=0}^{2} |\boldsymbol{q}|^{2-l} Z^{(l)}(\boldsymbol{q}^2)$$

➜ In order to compute the inclusive decay rate we need to solve the integral over ω.
Can we do this using lattice QCD correlators?

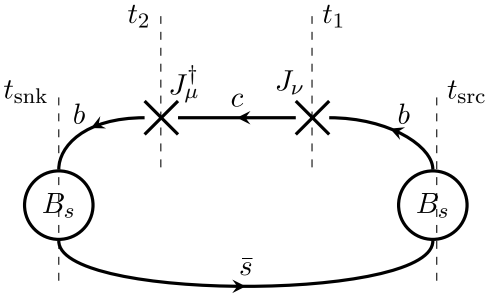
Decay rates from Euclidean correlators

On the lattice we can compute the forward-scattering matrix element from the 4-point function:

$$G_{\mu\nu}(t_2 - t_1, \boldsymbol{q}) = e^{-m_{B_s}|t_2-t_1|} \int d^3x \, \frac{e^{i\boldsymbol{q}\cdot\boldsymbol{x}}}{2m_{B_s}} \langle B_s(\mathbf{0})|J^\dagger_\mu(\boldsymbol{x}, t_2) J_\nu(\mathbf{0}, t_1)|B_s(\mathbf{0})\rangle$$

where:

$$G_{\mu\nu}(t_2 - t_1; \boldsymbol{q}) = \lim_{\substack{t_{\rm snk}\to+\infty \\ t_{\rm src}\to-\infty}} \frac{C_{\mu\nu}(t_{\rm snk}, t_2, t_1, t_{\rm src})}{C(t_{\rm snk} - t_2)C(t_1 - t_{\rm src})}$$



$C_{\mu\nu}(t_{\rm snk}, t_2, t_1, t_{\rm src})$

[Hashimoto, *PTEP*, 1703.01881 - Hansen, Meyer, Robaina, **PRD**, 1704.08993 - Gambino, Hashimoto, **PRL**, 2005.13730]

We can find a connection between lattice correlators and the hadronic tensor:

$$
\begin{aligned}
G_{\mu,\nu}(t_2 - t_1, \boldsymbol{q}) &= e^{-m_{B_s}|t_2-t_1|} \int d^3x e^{-i\boldsymbol{q}\cdot\boldsymbol{x}} \langle B(0)|J_\mu^\dagger(\boldsymbol{x}, t_2) J_\nu(\boldsymbol{0}, t_1)|B(0)\rangle \\
&= \int d^3x \; \langle B(0)|J_\mu^\dagger(0) e^{-\hat{H}|t_2-t_1| + (\hat{\boldsymbol{P}}-\boldsymbol{q})\cdot\boldsymbol{x}} J_\nu(0)|B(0)\rangle \\
&= \langle B(0)|J_\mu^\dagger(0) e^{-\hat{H}|t_2-t_1|} \delta^3(\hat{\boldsymbol{P}} - \boldsymbol{q}) J_\nu(0)|B(0)\rangle \\
&= \int d\omega \; e^{-\omega|t_2-t_1|} \langle B(0)|J_\mu^\dagger(0) \delta(\hat{H} - \omega) \delta^3(\hat{\boldsymbol{P}} - \boldsymbol{q}) J_\nu(0)|B(0)\rangle
\end{aligned}
$$

hence:

$$
\boxed{G_{\mu\nu}(t; \boldsymbol{q}) = \int_0^\infty d\omega \; W_{\mu\nu}(\omega, \boldsymbol{q}) \; e^{-\omega t}}
$$

Spectral densities & inverse problem

The problem of extracting $W_{\mu\nu}(\omega, \boldsymbol{q}^2)$ from 4-point correlators is equivalent to extracting $\rho(\omega)$ from 2-point correlators.

$$C(t) = \int_0^\infty d\omega \; \rho_L(\omega) \; K(\omega, t)$$

It requires solving an inverse problem which is ill-posed for lattice QCD correlators

This issue has been investigated for a long time especially in the context of finite temperature lattice QCD simulation

Currently there are several approaches to tackle the inverse problem:

- Backus-Gilbert
- Chebyshev Polynomials
- Bayesian inference (MEM, BR)
- Machine Learning/Neural Network, Gaussan Processes

Spectral reconstruction

The central idea is that we can calculate numerically

$$K(E,\omega) = \sum_{t=0}^{t_{\max}} g_t(E) e^{-\omega t}.$$

Then, spectral functions can be reconstructed applying linear combination of coefficients to LQCD correlators:

$$\hat{\rho}(E) = \int_0^\infty d\omega \; \rho_L(\omega) K(\omega, E) = \sum_{t=0}^{t_{\max}} g_t(E) \int_0^\infty d\omega \; \rho_L(\omega) e^{-\omega t}$$
$$\simeq \sum_{t=0}^{t_{\max}} g_t(E) C(t)$$

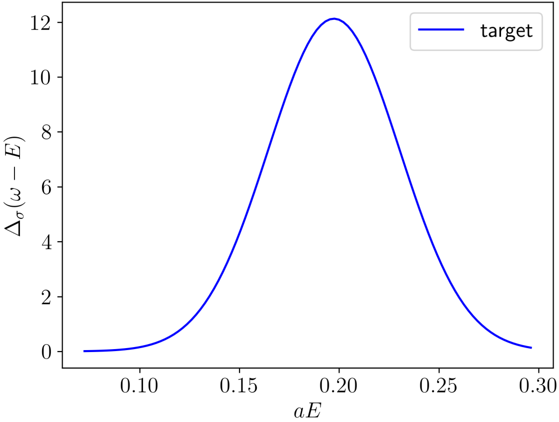
In an **ideal** scenario: $K(E,\omega) = \delta(\omega - E)$, so that

$$\hat{\rho}(E) = \int_0^\infty d\omega \; \delta(\omega - E) \rho_L(\omega)$$

Smeared kernels

In **practice**, we can numerically calculate only a $smeared$ version of the $\delta-$function.

$$\widehat{\rho}_{\sigma,L}(E) = \int_0^{\infty} d\omega \;\rho_L(\omega)\Delta_\sigma(E,\omega) \qquad\qquad \Delta_\sigma(E,\omega) = \sum_{t=0}^{t_{\max}} g_t(\sigma, E)\; e^{-a\omega t}$$



Smeared kernels

Following this recipe, we can extract the hadronic tensor:

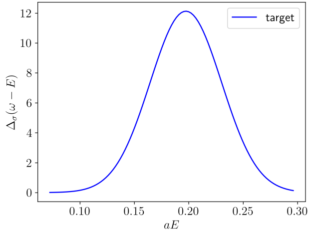
$$\widehat{W}^{(l)}_{\sigma,L}(E,\boldsymbol{q}^2) = \int_0^\infty d\omega\ W^{(l)}_L(\omega,\boldsymbol{q}^2) K_\sigma(E,\omega) \qquad K_\sigma(E,\omega) = \sum_{t=0}^{t_{\max}} g_t(\sigma,E)\ e^{-a\omega t}$$

choosing $K_\sigma(\omega) = \Delta_\sigma(\omega, E)$

$$\widehat{W}^{(l)}_{\sigma,L}(E,\boldsymbol{q}^2) = \int_0^\infty d\omega\ W^{(l)}_L(\omega,\boldsymbol{q}^2)\Delta_\sigma(\omega,E)$$

$$= \sum_t g_t(\sigma,\omega) \boxed{\int_0^\infty d\omega\ W^{(l)}_L(\omega,\boldsymbol{q}^2)\ e^{-a\omega t}}$$

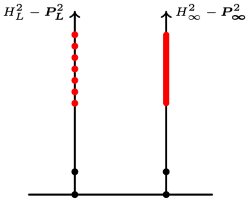
$$= \sum_t g_t(\sigma,\omega) G^{(l)}(at;\boldsymbol{q}^2)$$



$$W^{(l)}(E,\boldsymbol{q}^2) = \lim_{\sigma\to 0}\lim_{L\to\infty} \widehat{W}^{(l)}_{\sigma,L}(E,\boldsymbol{q}^2)$$

Comments on the limits

$$W^{(l)}(E, \boldsymbol{q}^2) = \lim_{\sigma \to 0} \lim_{L \to \infty} \widehat{W}^{(l)}_{\sigma, L}(E, \boldsymbol{q}^2)$$



- Smearing allow to replace $\sum_n \delta_n$ with a smooth function. This is necessary to perform a meaningful infinite volume limit.
  [Hansen,Meyer,Robaina, **PRD**, 1704.08993]

- $\sigma \to 0$ is not strictly necessary, **if** we want to compare with an experimental result which can be equally smeared as our spectral density.
  [Hansen,Lupo,Tantalo, **PRD**, 1903.06476]

Integration kernel

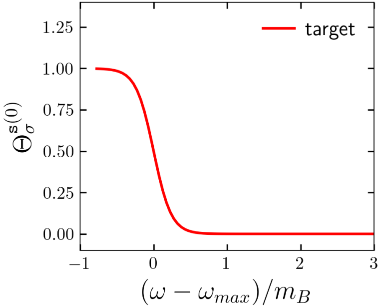
However, we can instead use $K_\sigma = \Theta^{(l)}_\sigma(\omega - \omega_{\max})$:

$$\Theta^{(l)}_\sigma(\omega - \omega_{\max}) = \sum_{t=0}^{t_{\max}} g_t(\sigma, \omega_{\max}) e^{-a\omega t}$$

Remember

$$Z^{(l)}(\boldsymbol{q}^2) = \int_0^\infty d\omega\ \Theta^{(l)}(\omega_{max} - \omega)\ W^{(l)}(\omega, \boldsymbol{q}^2)$$

?



smeared kernel

Integration kernel

However, we can instead use $K_\sigma = \Theta_\sigma^{(l)}(\omega - \omega_{\max})$:

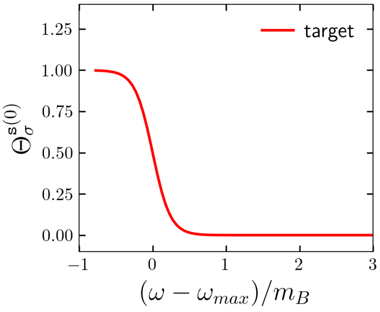
$$\Theta_\sigma^{(l)}(\omega - \omega_{\max}) = \sum_t^{t_{\max}} g_t(\sigma, \omega_{\max}) e^{-a\omega t}$$

$$\widehat{Z}^{(l)}_{\sigma,L}(\boldsymbol{q}^2) = \int_0^\infty d\omega\ \Theta_\sigma(\omega - \omega_{\max}) W_L^{(l)}(\omega, \boldsymbol{q}^2)$$

$$= \sum_t g_t(\sigma, \omega_{\max}) \boxed{\int_0^\infty d\omega\ W_L^{(l)}(\omega, \boldsymbol{q}^2)\ e^{-a\omega t}}$$

$$= \sum_t g_t(\sigma, \omega_{\max}) G^{(l)}(t, \boldsymbol{q}^2)$$

$$Z^{(l)}(\boldsymbol{q}^2) = \lim_{\sigma \to 0} \lim_{L \to \infty} \widehat{Z}^{(l)}_{\sigma,L}(\boldsymbol{q}^2, L)$$



smeared kernel

Workflow

We have a procedure to calculate inclusive rates directly from lattice correlators!

i. Calculate 4-point functions on the lattice $G^{(l)}(t, \boldsymbol{q}^2)$

ii. Approximate smeared kernel $\Theta_\sigma = \sum_t g_t(\sigma, \omega_{\max}) e^{-\omega t}$ to find g_t

iii. Calculate $\sum_t g_t(\sigma, \omega_{\max})\ G^{(l)}$ to obtain $\widehat{Z}^{(l)}_{\sigma,L}(\boldsymbol{q}^2)$

iv. Take the limits $\lim_{L\to\infty}$ and $\lim_{\sigma\to 0}$, in this order

I. Lattice correlators

ETMC

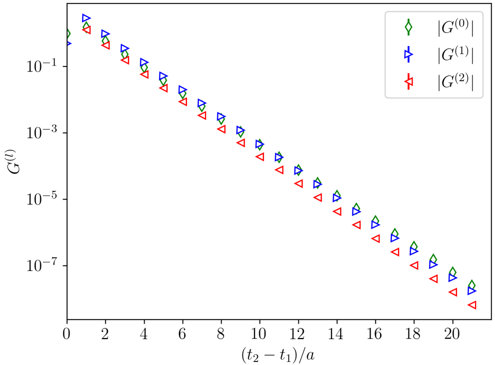
Twisted-Mass action (OS for s, c)

$N_f = 2+1+1,\ L \times T = 32^3 \times 64$

$a = 0.0815(30)\text{fm}, \quad M_\pi = 375(13)\ \text{MeV}$

$t_{\text{src}} = 0\ t_{\text{snk}} = 32a\ t_2 = 22a\ t_1 = 4a$

$m_b \simeq 2m_c \to M_{B_s} = 3.08(11)\text{GeV} < M_{B_s}^{phys}$



ETMC correlators
[Gambino,Hashimoto,Mächler,Panero,Sanfilippo, Simula,AS,Tantalo, JHEP, 2203.11762]

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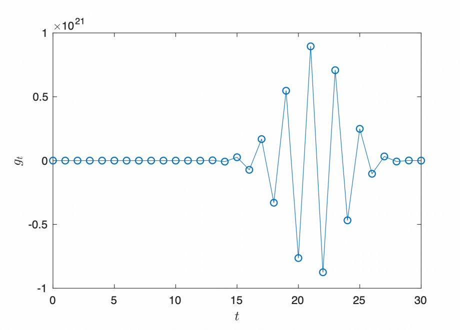
Backus-Gilbert regularisation

Hansen, Lupo, Tantalo, **PRD**, 1903.06476

$$A[g] = a \int_{E_0}^{\infty} dw \left\{ \Theta_\sigma^{(l)} - \sum_{t=1}^{t_{max}} g_t e^{-awt} \right\}^2$$



[Hansen, Lupo, Tantalo, **PRD**, 1903.06476]

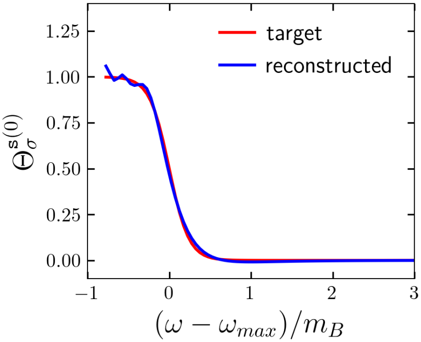
Backus-Gilbert regularisation

Hansen, Lupo, Tantalo **PRD**, 1903.06476

$$W_\lambda[g] = (1-\lambda)\frac{A[g]}{A[0]} + \lambda B[g], \quad \left.\frac{\partial W_\lambda[g]}{\partial g_t}\right|_{g_t = g_t^\lambda} = 0$$

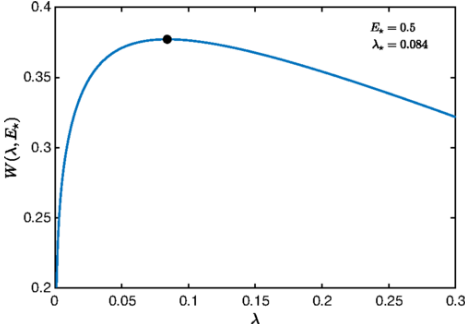
$$A[g] = a\int_{E_0}^{\infty} dw \left\{ \Theta_\sigma^{(l)} - \sum_{t=1}^{t_{max}} g_t e^{-awt} \right\}^2$$

$$B[g] = \sum_{t,t'=1}^{t_{max}} g_t g_{t'} \frac{\mathrm{Cov}[G^{(l)}(at), G^{(l)}(at')]}{[G^{(l)}(0)]^2}$$



[Gambino,Hashimoto,Mächler, Panero,Sanfilippo,Simula,AS,Tantalo, JHEP, 2203.11762]

Finding $\lambda_\star$



[Hansen, Lupo, Tantalo, **PRD**, 1903.06476]

$$\left.\frac{\partial W(\lambda)}{\partial \lambda}\right|_{\lambda=\lambda_\star} = 0$$

- small λ: good approximation **but** large statistical errors (inverse problem)
- large λ: bad approximation **but** small statistical errors (eccessive regularisation)
- $\lambda_\star$: optimal balance between systematic and statistical errors

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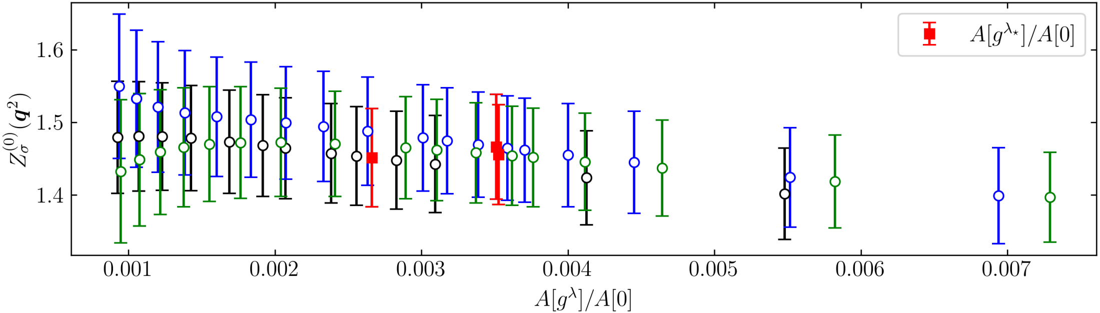
iii. calculate $\sum_t g_t(\sigma, \omega_{\max})\ G^{(l)}$ to obtain $\widehat{Z}^{(l)}_{\sigma,L}(\boldsymbol{q}^2)$

iv. take the limit $\lim_{L\to\infty}$ and then $\lim_{\sigma\to 0}$, in this order

III. calculate $\widehat{Z}^{(l)}_{\sigma,L}(\boldsymbol{q}^2)$ Bulava, Hansen M.T., Hansen M.W., Patella, Tantalo, *JHEP*, 2111.12774



[Gambino,Hashimoto,Mächler,Panero,Sanfilippo,Simula,AS,Tantalo, JHEP, 2203.11762]

$$\theta^{\rm s}_\sigma(x) = \frac{1}{1+e^{-\frac{x}{\sigma}}}, \qquad \theta^{\rm s1}_\sigma(x) = \frac{1}{1+e^{-\sinh(\frac{x}{r^{\rm s1}\sigma})}}, \qquad \theta^{\rm e}_\sigma(x) = \frac{1+\mathrm{erf}(\frac{x}{r^{\rm e}\sigma})}{2}$$

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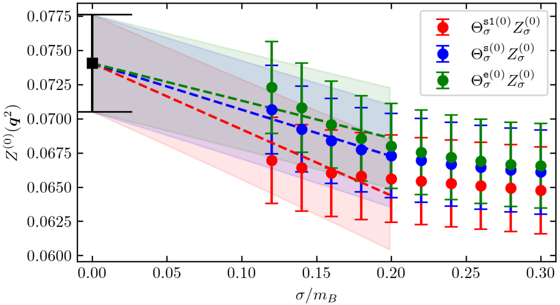
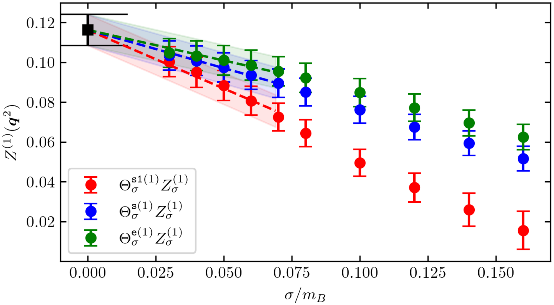
IV. extrapolate to $\sigma \to 0$





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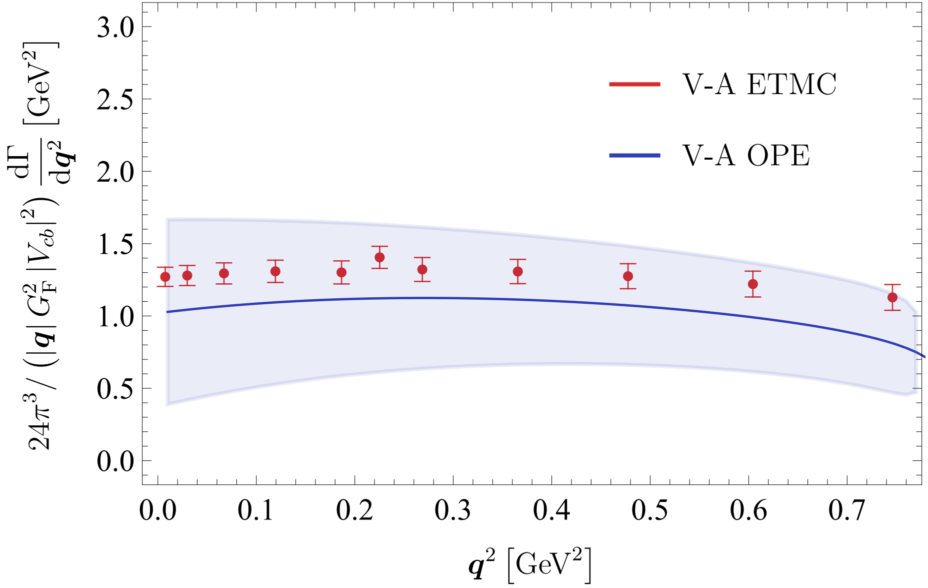
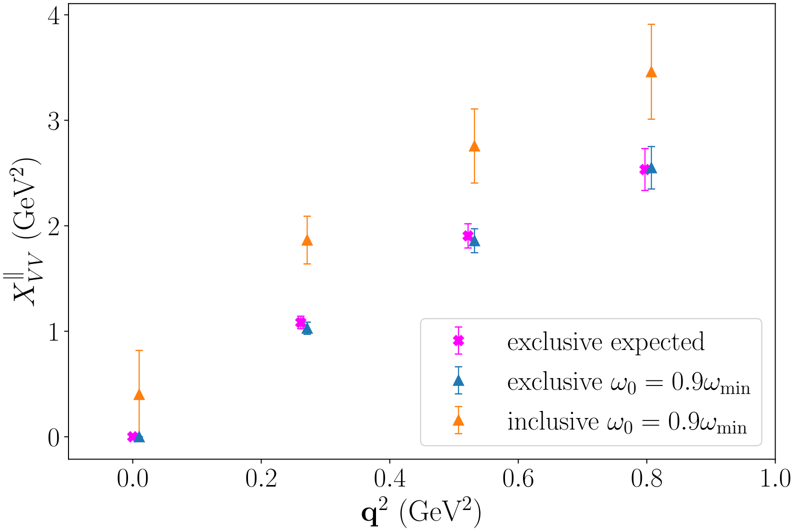
Comparison with OPE



[Gambino,Hashimoto,Mächler,Panero,Sanfilippo,Simula,AS,Tantalo, JHEP, 2203.11762]

Comaprison with exclusive decays



[Barone,Hashimoto,Jüttner,Kaneko,Kellermann, JHEP, 2203.11762]

Conclusions

- Inclusive semi-leptonic decays are now accessible with lattice QCD
- One can use different methods to perform the kernel reconstruction and perform the numerical integration of the structure functions (Chebyshev, Backus-Gilbert, ...)
- Agreement between lattice QCD results and OPE is encouraging
- For physically relevant results, need to take all the relevant extrapolations: $a \to 0$, $L \to \infty$
- simulations with m_b^{phys} necessary for accessing the correct phase space
- Method is general and can be applied to $B \to X_u l\nu$ and $D \to Xl\nu$ decays as well as $\tau \to X_{ud}\nu_t$

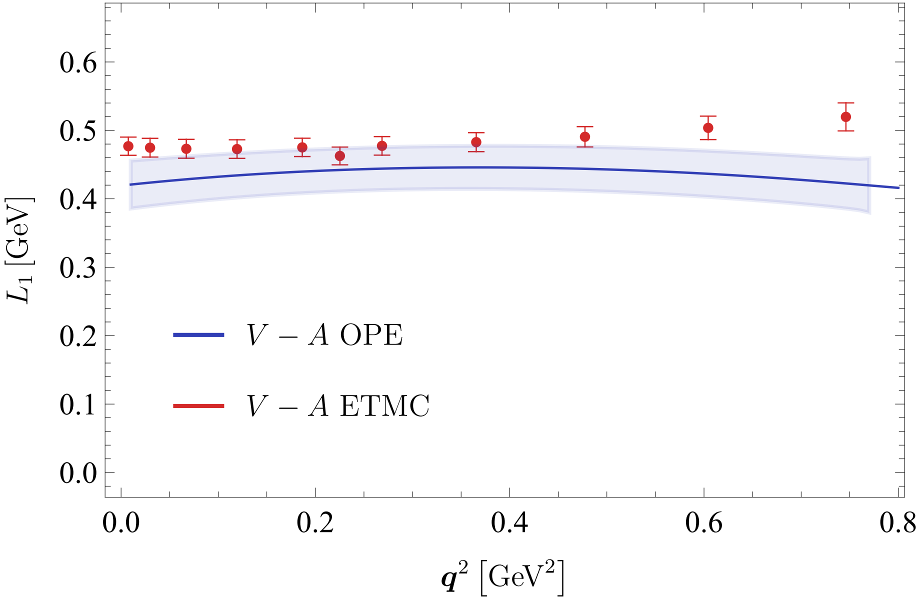
BACKUP SLIDES

Lepton energy moments

$$L_{n_\ell}(\boldsymbol{q}^2) = \frac{\int dq_0 dE_\ell \; E_\ell^{n_\ell} \left[\frac{d\Gamma}{d\boldsymbol{q}^2 dq_0 dE_\ell}\right]}{\int dq_0 dE_\ell \left[\frac{d\Gamma}{d\boldsymbol{q}^2 dq_0 dE_\ell}\right]}$$

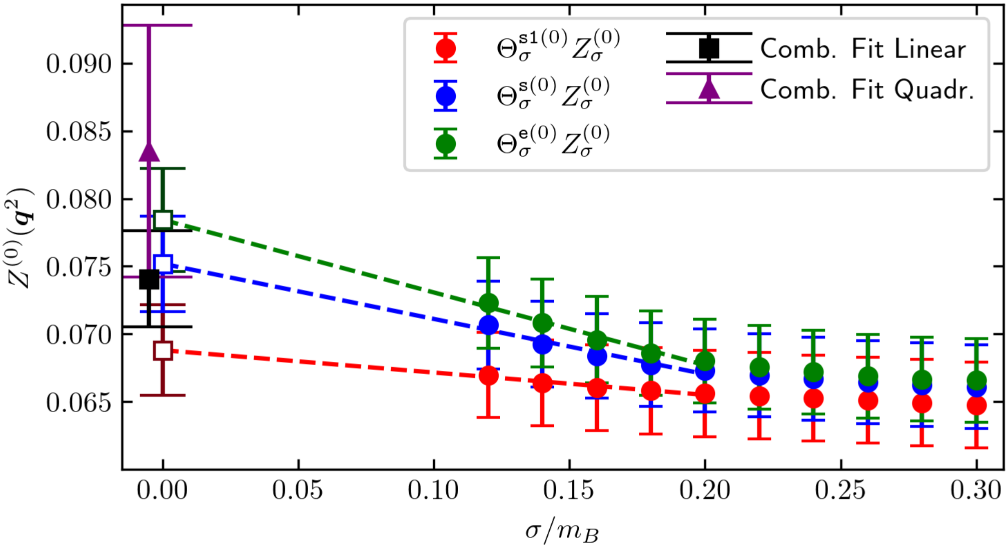
$$\overline{Z}_{n_\ell=1}(\boldsymbol{q}^2) = \sum_{l=0}^{3} (\sqrt{\boldsymbol{q}^2}) Z^{(l)}_{n_\ell=1}(\boldsymbol{q}^2)$$

$$Z^{(l)}_{n_\ell=1}(\boldsymbol{q}^2) = \int_0^\infty d\omega \; \Theta^{(l)}(\omega_{\max-\omega}) W^{(l)}_{n_\ell=1}(\omega, \boldsymbol{q}^2)$$

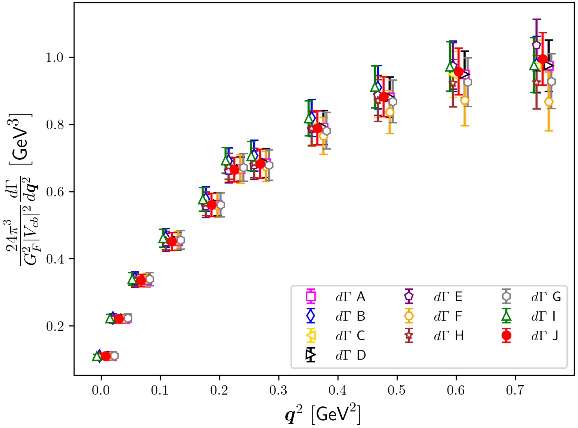


L_1 [GeV]
0.6
0.5
0.4
0.3
0.2
0.1
0.0
0.0
0.2
0.4
0.6
0.8
$\boldsymbol{q}^2$ [GeV^2]
$V - A$ OPE
$V - A$ ETMC

Extrapolation



Systematics



(a) extrapolate $Z^{(l)}$ individually and then sum

(b) extrapolate with all values of σ

(c) choosing $\lambda < \lambda_\star$

(d) choosing $\lambda > \lambda_\star$

(e) sum and the extrapolate sum of $Z^{(l)}$

(f) $\tau_{\max} = 15$

(g) $\tau_{\max} = 16$

(h) $\tau_{\max} = 17$

(i) Bootstrap

Tensor Decomposition

Accoding to Lorentz invariance and time-reversal symmetry, the Hadronic Tensor can be decomposed as follows

$$W^{\mu\nu}(p,q) = -g^{\mu\nu}W_1(w,\boldsymbol{q}^2) + \frac{p^\mu p^\nu}{m_{B_s}^2}W_2(w,\boldsymbol{q}^2) - \frac{i\varepsilon^{\mu\nu\alpha\beta}p_\alpha q_\beta}{m_{B_s}^2}W_3(w,\boldsymbol{q}^2) + \frac{q^\mu q^\nu}{m_{B_s}^2}W_4(w,\boldsymbol{q}^2) + \frac{p^\mu q^\nu + p^\nu q^\mu}{m_{B_s}^2}W_5(w,\boldsymbol{q}^2)$$

For convenience we will redefine these components w.r.t. a different basis:

$$\hat{\boldsymbol{n}} = \frac{\boldsymbol{q}}{\sqrt{\boldsymbol{q}^2}} \qquad \boldsymbol{\varepsilon}^{(a)} \cdot \hat{\boldsymbol{n}} = 0 \qquad \boldsymbol{\varepsilon}^{(a)} \cdot \boldsymbol{\varepsilon}^{(b)} = \delta^{ab}$$

$$Y^{(1)} = -\sum_{a=1}^{2} \sum_{i,j=1}^{3} \varepsilon_i^{(a)} \varepsilon_j^{(a)} W^{ij}$$

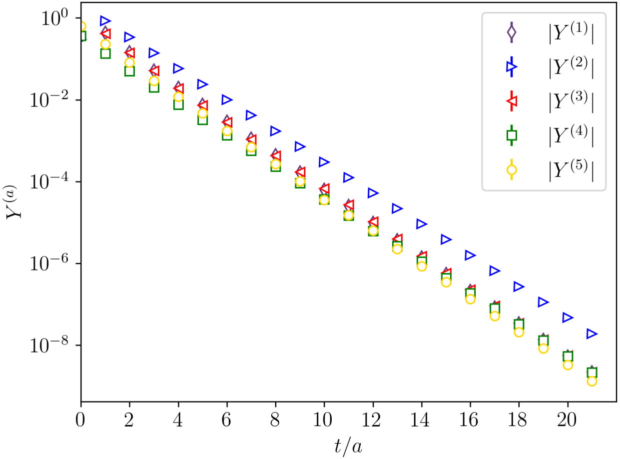
$$Y^{(2)} = W^{00}$$

$$Y^{(3)} = \sum_{i,j=1}^{3} \hat{n}^i \hat{n}^j W^{ij}$$

$$Y^{(4)} = \sum_{i=1}^{3} \hat{n}^i (W^{0i} + W^{i0})$$

$$Y^{(5)} = \frac{i}{2} \sum_{i,j,k=1}^{3} \epsilon^{ijk} \hat{n}^k W^{ij}$$

Y correlators



Decay rate structure functions

$$W^{(0)} = Y^{(2)} + Y^{(3)} - Y^{(4)}$$

$$W^{(1)} = 2Y^{(3)} - 2Y^{(1)} - Y^{(4)}$$

$$W^{(2)} = Y^{(3)} - Y^{(1)}$$

This presentation is mainly based on:
"*Lattice QCD study of inclusive semileptonic decays of heavy mesons*"
J. High Energ. Phys. 2022, 83 (2022) [hep-lat: 2203.11762]

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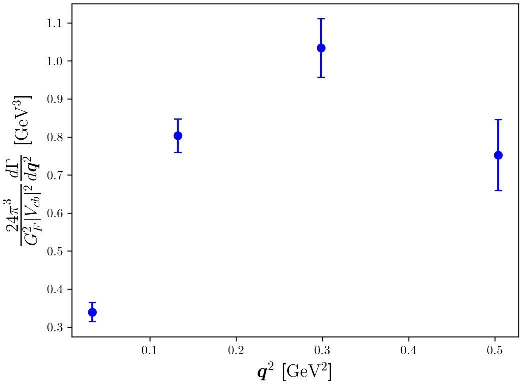
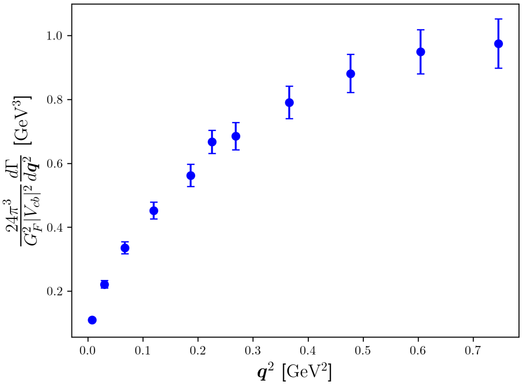
Inclusive decay rate



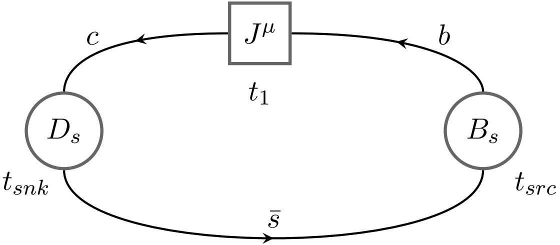
$D_s \to X l \nu$



$B_s \to X_c l \nu$

Lattice QCD and semileptonic B-decays

Lattice QCD has been extremely succesful in determining $|V_{cb}|$ to a very high precision through the precise calculation of form factors, $e.g.$:



$$\frac{\langle D_s(p')|V^\mu|B_s(p)\rangle}{\sqrt{M_{B_s}M_{D_s}}} = h_+(w)(v+v')^\mu + h_-(w)(v-v')^\mu$$

where h_+ and h_- can be extracted from fits of 2- and 3-point correlation functions and can be used to calculate the relevant form factor.

Lattice QCD and semileptonic B-decays

The study of exclusive semileptonic decays on the lattice requires information only about the **ground state** of the daughter meson, which is something which is easily accessible in lattice QCD.

➜ Things are different for inclusive semileptonics decays, where we require a sum over all final states $\sum X$