Hidden Conformal Symmetry from the Lattice

PRD **108**, L091505

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Outline

Collaborators

Lattice Strong Dynamics

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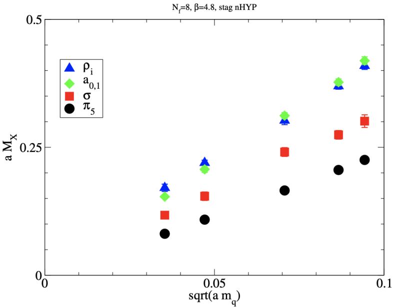
The $N_f = 8$ Theory

We analyze our latest lattice data for the $\mathrm{SU}(3)$ gauge theory with 8 Dirac fermion flavors. The data is presented in 2306.06095:



- This gauge theory is believed to lie close to the boundary of the conformal window.
- The σ and π are somewhat separated from the ρ in the spectrum.

$\mathrm{SU}(3)$ with $N_f = 8$ can be used to build composite Higgs models, e.g PRL **126** (2021) 191804

Dilaton EFT

Reviewed in Universe 9 (2023) 1, 10 with T. Appelquist and M. Piai.

Field Content

1. $N_f^2 - 1$ NGB fields π^a
   $\Sigma = \exp\{2i\pi^a T^a / F_\pi\}$
   $\langle\Sigma\rangle = \mathbb{1}$
2. Dilaton field χ
   $\langle\chi\rangle = F_d$

See dilaton EFT of Golterman and Shamir: PRD **94** (2016)

Symmetries

Chiral Symmetry

$$\mathrm{SU}(N_f)_L \times \mathrm{SU}(N_f)_R \to \mathrm{SU}(N_f)_V$$
$$\Sigma \to L\Sigma R^\dagger$$

Scale Invariance

$$\text{Scale} \times \text{Poincaré} \to \text{Poincaré}$$
$$\chi(x) \to e^\lambda \chi(e^\lambda x)$$

Dilaton EFT at Leading Order

Theory Lagrangian

$$\mathcal{L} = \frac{1}{2}\left(\partial_\mu \chi\right)^2 + \frac{f_\pi^2}{4}\left(\frac{\chi}{f_d}\right)^2 \mathrm{Tr}\left[\partial_\mu \Sigma \partial^\mu \Sigma^\dagger\right] + \frac{m B_\pi f_\pi^2}{2}\left(\frac{\chi}{f_d}\right)^y \mathrm{Tr}\left[\Sigma + \Sigma^\dagger\right] - V(\chi)\,. \quad (1)$$

- NGB terms are similar to those in chiral Lagrangian.
- Dependence on compensator field χ is determined by scale invariance.
- Expect $f_\pi \sim f_d$ set by confinement scale.
- Parameter y has been identified with scaling dimension of $\bar{\psi}\psi$ above the confinement scale: Bardeen et al NPB **323**, 493 (1989).

Dilaton Potential

$$V_\Delta = \frac{m_d^2 \chi^4}{4(4-\Delta) f_d^2}\left[1 - \frac{4}{\Delta}\left(\frac{f_d}{\chi}\right)^{4-\Delta}\right]. \quad (2)$$

- Potential contains a scale invariant term ($\sim \chi^4$) and a deformation ($\sim \chi^\Delta$), which explicitly violates scale invariance.
- This potential has a minimum at $\chi = f_d$, and a weak curvature $m_d^2 \ll (4\pi f_d)^2$.
- For $\Delta < 4$, V_Δ grows as χ^4 for large χ.
- For $\Delta > 4$, V_Δ grows as χ^Δ for large χ.
- Potentials of this form are discussed in e.g: Rattazzi & Zaffaroni JHEP **0104**, 021 (2001), GGS PRL.**100** 111802, (2008), CCT PRD.**100** 095007 (2019).

Dilaton Potential

Special case: The SM Higgs potential $\Delta = 2$.

$$V(\chi) = \frac{m_d^2}{8f_d^2}\left(\chi^2 - f_d^2\right)^2 \qquad (3)$$

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Special case: Near marginal deformation $\Delta \to 4$.

$$V(\chi) = \frac{m_d^2}{16f_d^2}\chi^4\left(4\ln\frac{\chi}{f_d} - 1\right) \tag{4}$$

Scalar Decay Constant

Measured by LatKMI in PRD 96 (2017) 014508

Define scalar decay constant using the matrix element

$$\langle 0|\, J_S(x)\, |\chi(p)\rangle \equiv F_S M_d^2 e^{-p\cdot x}\,, \tag{5}$$

where

$$J_S(x) \equiv m \sum_{i=1}^{N_f} \bar{\psi}_i \psi_i\,. \tag{6}$$

1. F_S can be extracted from lattice measurement of correlator $\langle J_S(x) J_S(0)\rangle$, which is used already to measure M_d.
2. It is a true decay constant: It would control the decay rate of the dilaton if there was a heavy scalar mediator coupled to $\bar{\psi}\psi$ along with light states. Analogous to f_π for the QCD pion decaying to leptons via $W^{\pm}$.

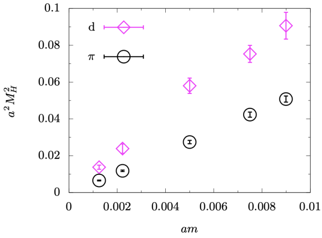
Scalar Decay Constant

This quantity can also be calculated in dilaton EFT:

$$|F_S| = \frac{yN_f M_\pi^2 F_\pi}{2M_d^2} \frac{f_\pi}{f_d}. \tag{7}$$

- Incorporating Eq. (7) into our EFT fit provides a direct test of the coupling between the light scalar and the fermion mass, treated as an external source.

Lattice Data



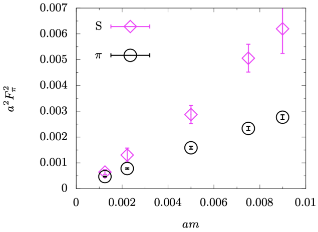


Figure: Lattice data for M_π^2, M_d^2, F_π^2 and F_S^2 from LSD 2306.06095. The lattice spacing is denoted by a.

We also include data for the π–π scattering length in the I=2, $\ell = 0$ channel from LSD PRD **105** (2022) 034505

Result Of Global Fit to dEFT

Parameter	Value and Uncertainty
y	2.091(32)
aB_π	2.45(13)
Δ	3.06(41)
$a^2 f_\pi^2$	$6.1(3.2) \times 10^{-5}$
f_π^2 / f_d^2	0.1023(35)
m_d^2 / f_d^2	1.94(65)
χ^2/dof	21.3/19=1.12

Table: Central values of fit parameters obtained in a six parameter global fit to LSD data for $M^2_{\pi,d}$, $F^2_{\pi,S}$ and scattering length.

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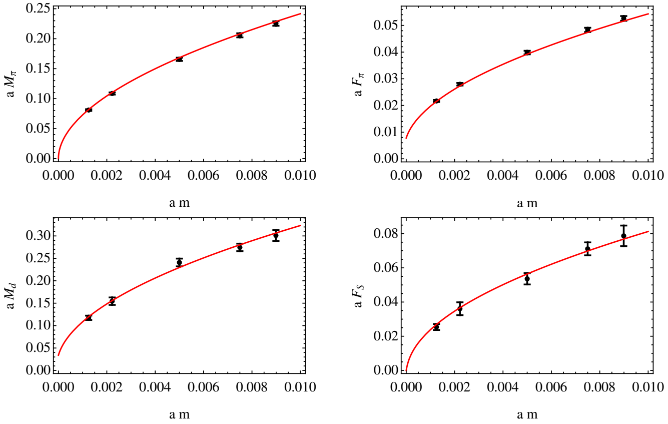
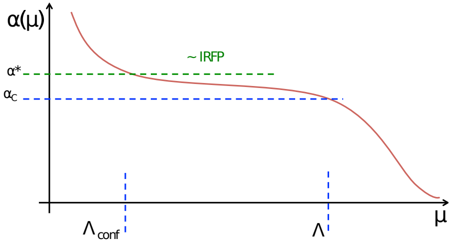


Interpretation of Δ



1. Strongly coupled over large interval of scales $\Longrightarrow$ possibility of large anomalous dimensions. Note we found $y \approx 2$.

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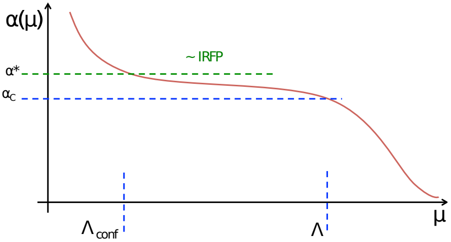

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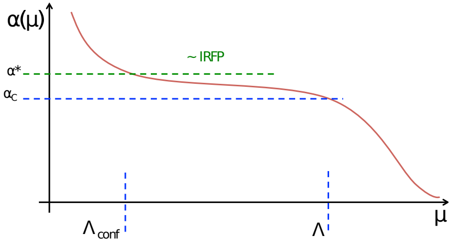

1. Strongly coupled over large interval of scales $\implies$ possibility of large anomalous dimensions. Note we found $y \approx 2$.
2. Allows for new relevant interactions besides (near marginal) gauge interaction.
3. Δ should be identified with the engineering plus anomalous dimension of this new relevant operator.

Scaling Relations at Leading Order

We also want to test the alternate possibility - that the $N_f = 8$ theory is *inside* the conformal window.

Assuming the gauge coupling g has reached its fixed point value $g^\star$, physical quantities may be fitted to scaling relations Zwicky, del Debbio PLB **700** (2011)

$$M_X = C_X m^{[1/(1+\gamma^\star)]}, \tag{8}$$

$$F_Y = C_Y m^{[1/(1+\gamma^\star)]}, \tag{9}$$

$$1/a_0^{(2)} = C_a m^{[1/(1+\gamma^\star)]}. \tag{10}$$

Following approach of Appelquist et al PRD **84** (2011) 054501.

Result of Global Fit to Mass–Deformed CFT

Fitting to the same set of lattice data as in the dilaton case, we find:

Parameter	Value and Uncertainty
C_{M_π}	2.121(78)
C_{F_π}	0.522(19)
C_{M_d}	2.97(12)
C_{F_S}	0.706(33)
C_a	-5.88(22)
γ^*	1.073(28)
χ^2/dof	$48.1/19 = 2.53$

The χ^2/dof is larger than for the dEFT fit, while the number of fit parameters is the same. This indicates a lower quality fit.

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- Assuming the gauge theory is inside the conformal window, we fit the same set of lattice data to mass–deformed CFT scaling relations. This fit is of lesser quality.
- The worse mdCFT fit could be a consequence of $g \not\approx g^\star$.
- Adding particular NLO corrections can improve the AIC for both kinds of fit. The required NLO corrections are large in the mdCFT case.

Thank you!

Lattice Action

- Our numerical calculations use improved nHYP smeared staggered fermions with smearing parameters $\alpha = (0.5, 0.5, 0.4)$. [LSD PRD 99(2019)014509]
- $\beta_A/\beta_F = -0.25$ where $\beta_F = 4.8$.
- After taste splitting, only $\mathrm{SU}(2)_L \times \mathrm{SU}(2)_R$ flavor symmetry preserved in massless theory (3 exact NGBs).
- Spectral study has revealed that the taste splitting of the 63-plet masses are on the order of 20–30%. [LSD PRD 99(2019)014509]

Summary of Improvements to Lattice Dataset

Presented in 2306.06095

Since the previous LSD study of the $N_f = 8$ theory PRD **99** (2019) 014509, we have made some changes.

1. We have data for a new observable: The scalar decay constant F_S.
2. We have extrapolated the quantities M_π, F_π, M_σ (and also F_S) to the infinite volume limit.
3. We have improved our estimates of systematic uncertainties using Bayesian Model Averaging Jay, Neil PRD **103** (2021) 114502

The $N_f = 8$ spectrum has also been calculated before in LatKMI PRD **96** (2017) 014508

$I = 2$ Interpolating Operators

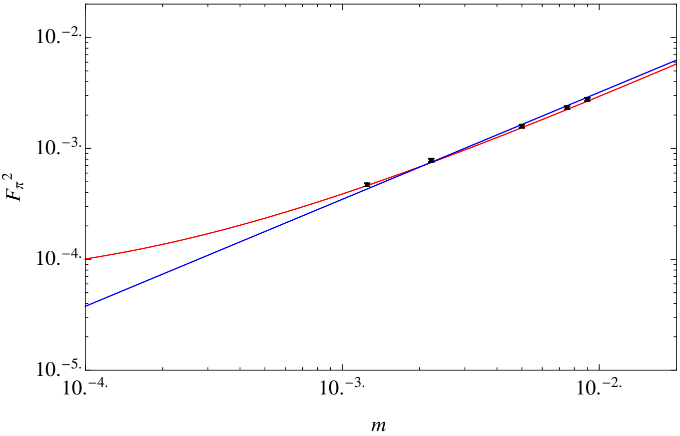
$$\pi^+(t) = \sum_{\vec{x}} \bar{\chi}_2(x)\epsilon(x)\chi_1(x), \text{ where } \epsilon(x) = (-1)^{x+y+z+t} \tag{11}$$

$$\mathcal{O}_{I=2}(t) = \pi^+(t)\pi^+(t+1) \tag{12}$$

$$\begin{aligned} C_{I=2}(t, t_0) &= \langle \mathcal{O}_{I=2}(t)\mathcal{O}_{I=2}(t_0)^\dagger \rangle \\ &= \sum_{\vec{x}_1,\cdots,\vec{x}_4} \langle \pi^+(t_4,\vec{x}_4)\pi^+(t_3,\vec{x}_3)\pi^+(t_2,\vec{x}_2)^\dagger \pi^+(t_1,\vec{x}_1)^\dagger \rangle \end{aligned} \tag{13}$$

Wall sources used - moving wall method.

Extrapolation Of F_π^2

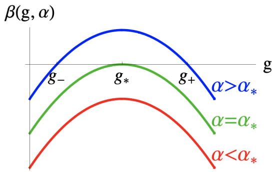


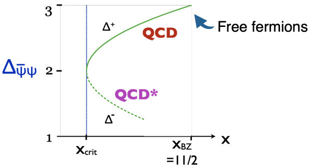
Marginality Crossing

Gies and Jaeckel: Eur.Phys.J.C46 (2006)

Kaplan, Lee, Son and Stephanov: Phys.Rev.D80 (2009)

Gukov: Nucl.Phys.B.919 (2017)





$$\mathcal{L} = \frac{1}{4}\mathrm{Tr}\left[G_{\mu\nu}G^{\mu\nu}\right] + \sum_i \bar{\psi}_i \not{D} \psi_i + \mathcal{L}_{4\text{ fermi}} \tag{14}$$

The conformal window is exited when a 4 fermi operator becomes relevant.

New Relevant Operators

There are 4 independent chiral symmetry preserving 4 fermi operators in $\mathrm{SU}(N_c)$ gauge theory with N_f Dirac fermions.

$$\mathcal{L}_{4\text{ fermi}} = \sum_{i=1}^{4} c_i \mathcal{O}_i(x) \tag{15}$$

$$\begin{aligned}
\mathcal{O}_1 &= \overline{\psi}_i \gamma^\mu \psi^j \overline{\psi}_j \gamma_\mu \psi^i + \overline{\psi}_i \gamma^\mu \gamma_5 \psi^j \overline{\psi}_j \gamma_\mu \gamma_5 \psi^i \\
\mathcal{O}_2 &= \overline{\psi}_i \psi^j \overline{\psi}_j \psi^i - \overline{\psi}_i \gamma_5 \psi^j \overline{\psi}_j \gamma_5 \psi^i \\
\mathcal{O}_3 &= (\overline{\psi}_i \gamma^\mu \psi^i)^2 - (\overline{\psi}_i \gamma^\mu \gamma_5 \psi^i)^2 \\
\mathcal{O}_4 &= (\overline{\psi}_i \gamma^\mu \psi^i)^2 + (\overline{\psi}_i \gamma^\mu \gamma_5 \psi^i)^2
\end{aligned}$$

We identify Δ with the scaling dimension of the relevant operator.

Corrections to Scaling in mdCFT

If we continue to assume that $g \approx g^\star$, we can also expect corrections to scaling relations that are polynomial in m.

Adding the next-to-leading corrections yields

$$M_X = C_X m^{[1/(1+\gamma^\star)]} + D_X m \,, \tag{16}$$

$$F_Y = C_Y m^{[1/(1+\gamma^\star)]} + D_Y m \,, \tag{17}$$

$$1/a_0^{(2)} = C_a m^{[1/(1+\gamma^\star)]} + D_a m \,. \tag{18}$$

To compare the quality of fits to models with different numbers of free parameters, we use the Akaike Information Criterion (AIC). Models with lower AIC are more probable in a Bayesian framework.

NLO Fits to mdCFT

We have only added the correction terms which minimize the AIC.

Parameter	LO	NLO 1	NLO 2
C_{M_π}	2.121(78)	1.56(11)	1.57(12)
C_{F_π}	0.522(19)	0.445(21)	0.448(23)
C_{M_d}	2.97(12)	2.53(12)	2.55(13)
C_{F_S}	0.706(33)	0.599(33)	0.459(63)
C_a	-5.88(22)	-5.05(24)	-5.86(53)
γ^*	1.073(28)	1.207(41)	1.200(44)
D_{M_π}	—	4.80(87)	4.71(90)
D_{F_S}	—	—	2.77(98)
D_a	—	—	12.9(5.8)
χ^2/dof	48.1/19	20.9/18	6.90/16
AIC	60.1	34.9	24.9

NLO correction to F_S grows to 46% of LO contribution size.

NLO Corrections in dEFT

We do not have complete NLO calculations for all our observables in dEFT.

Some of these corrections will likely come suppressed by $M_\pi^2/(4\pi F_\pi)^2$.

Lets take a phenomenological approach and add a contribution to the observable that shows the largest tension in the fit:

$$M_\pi a_0^{(2)} = \frac{-M_\pi^2}{16\pi F_\pi^2}\left(1-(y-2)^2\frac{f_\pi^2}{f_d^2}\frac{M_\pi^2}{M_d^2}+\frac{l_a M_\pi^2}{(4\pi F_\pi)^2}\right), \tag{19}$$

We neglect potential chiral logs.

NLO Fit in dEFT

Parameter	LO	NLO
y	2.091(32)	2.069(32)
B_π	2.45(13)	2.46(13)
Δ	3.06(41)	2.88(49)
f_π^2	$6.1(3.2) \times 10^{-5}$	$5.8(3.4) \times 10^{-5}$
f_π^2 / f_d^2	0.1023(35)	0.1089(41)
m_d^2 / f_d^2	1.94(65)	2.24(80)
l_a	—	0.78(27)
χ^2/dof	21.3/19	10.3/18
AIC	33.3	24.3

The AIC is reduced by adding the NLO correction to a level below the AIC in the NLO mdCFT case.

Correction is small - under 10%.