Lattice QCD calculations for Muon g-2

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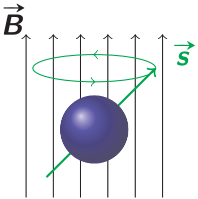
Magnetic Moment of the Muon

- magnetic moment $\vec{\mu}$ of the muon due to its spin $\vec{s}$ and electric charge e

$$\vec{\mu} = g \frac{e}{2m} \vec{s}$$

torque $\vec{\tau} = \vec{\mu} \times \vec{B}$



- gyromagnetic-factor (g-factor) of the muon

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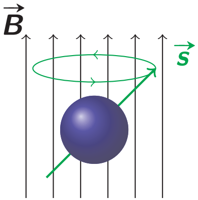
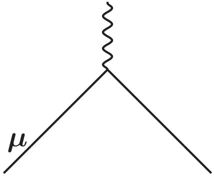


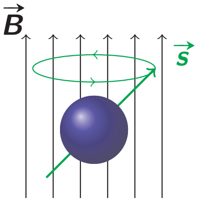
- gyromagnetic-factor (g-factor) of the muon

without quantum effects:

$$g = 2$$



Magnetic Moment of the Muon



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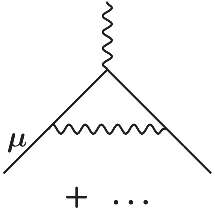
- gyromagnetic-factor (g-factor) of the muon

with quantum effects:

$$g = 2.00233\ldots$$

anomalous magnetic moment of the muon "Muon g-2"

$$a_\mu = \frac{g-2}{2}$$

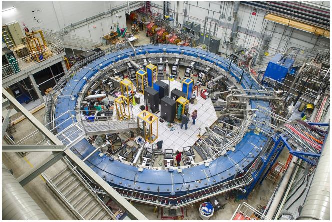


Muon g-2: Experimental measurement

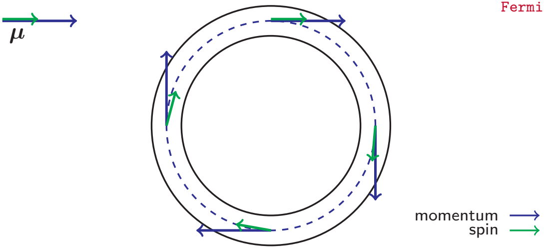
Previous: Muon g-2 @ BNL (2006)
[Phys.Rev. D73, 072003 (2006)]

New: Muon g-2 @ FNAL (2021)
[PhysRevLett.126.141801 (2021)]

measure precession frequency of muons in magnetic field:



[https://commons.wikimedia.org/wiki/File:Fermilab_g-2_(E989)_ring.jpg]



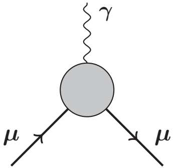
$$\omega_a = a_\mu \frac{eB}{m_\mu}$$

$$a_\mu(\exp) = 11659206.1(4.1) \times 10^{-10}$$

[PhysRevLett.126.141801 (2021)]

Muon g-2: Standard Model Prediction

White Paper (2020) of the Muon g-2 Theory initiative
[Phys.Rept. 887 (2020) 1-166]
[https://muon-gm2-theory.illinois.edu/]



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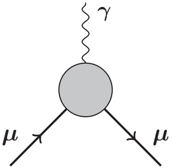

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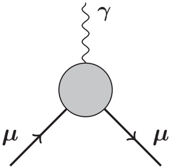
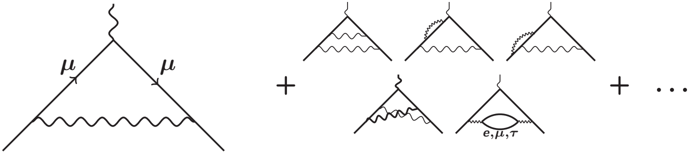


electro-magnetism

$\mathbf{11658471.8931(104) \times 10^{-10}}$



$O(10^4)$ diagrams at $O(\alpha^5)$

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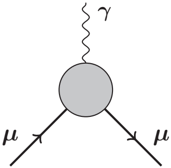
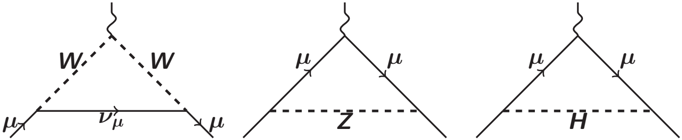


electro-magnetism	$\mathbf{11658471.8931(104)} \times \mathbf{10^{-10}}$
weak	$\mathbf{15.36(10)} \times \mathbf{10^{-10}}$



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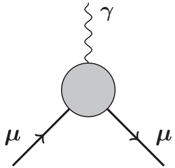
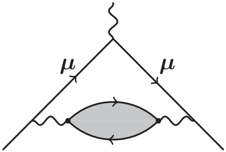


electro-magnetism	$\mathbf{11658471.8931(104)} \times \mathbf{10^{-10}}$
weak	$\mathbf{15.36(10)} \times \mathbf{10^{-10}}$
Hadronic Vacuum Polarisation (HVP)	$\mathbf{693.1(4.0)} \times \mathbf{10^{-10}}$



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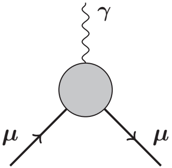

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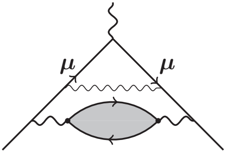
HVP(α^3,α^4)	$\mathbf{-8.59(7)} \times \mathbf{10^{-10}}$



$+ \quad \ldots$

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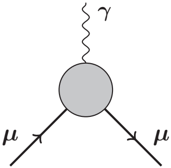

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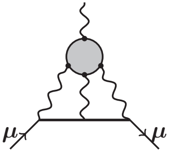
Hadronic light-by-light scattering	$\mathbf{9.2(1.8)} \times \mathbf{10^{-10}}$



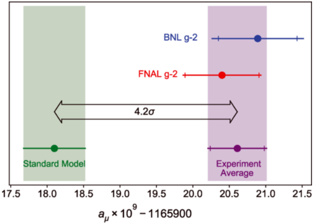
Experiment vs Standard Model prediction

Exp: $a_\mu = 0.00116592061(41)$

SM: $a_\mu = 0.00116591810(43)$

- This could be new physics!

Muon g−2 Coll., Phys. Rev. Lett. 126, 141801



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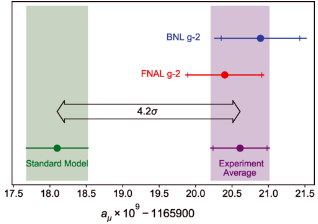

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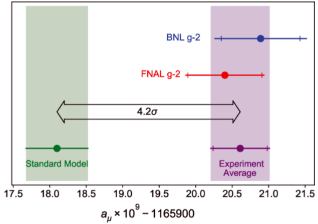
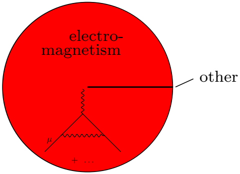


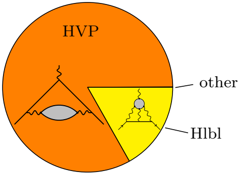
What's next?

- FNAL reduce error by factor $\sim \mathbf{4}$, new upcoming experiment @JPARC
- Breakdown of Standard Model Prediction

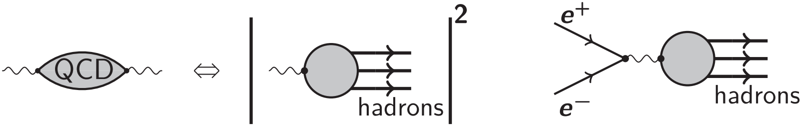
contribution to $\boldsymbol{a_\mu}$



contribution to variance $\boldsymbol{\Delta^2 a_\mu}$

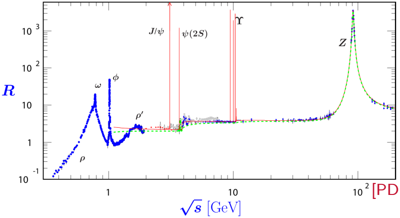


The HVP from R-ratio



$$R(s) = \frac{\sigma(e^+ e^- \to \text{hadrons}, s)}{\sigma(e^+ e^- \to \mu^+\mu^-, s)}$$

$$a_\mu^{\text{HVP}} = \left(\frac{\alpha m_\mu}{3\pi}\right)^2 \int_{m_\pi^2}^{\infty} \mathrm{d}s \, \frac{R(s)K(s)}{s^2}$$



[PDG]

$a_\mu^{\text{hvp}} = 689.46(3.25)$ [Jegerlehner 18]

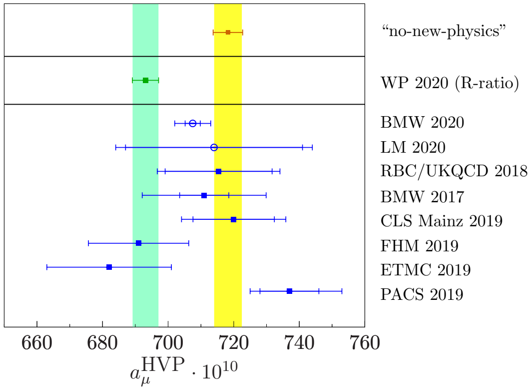
$a_\mu^{\text{hvp}} = 693.9(4.0)$ [DHMZ 19]

$a_\mu^{\text{hvp}} = 693.37(2.46)$ [KNT 18]

$a_\mu^{\text{hvp}} = 693.1(4.0)$ [white paper]

Lattice calculation of HVP

- Comparision of available lattice QCD calculations of HVP



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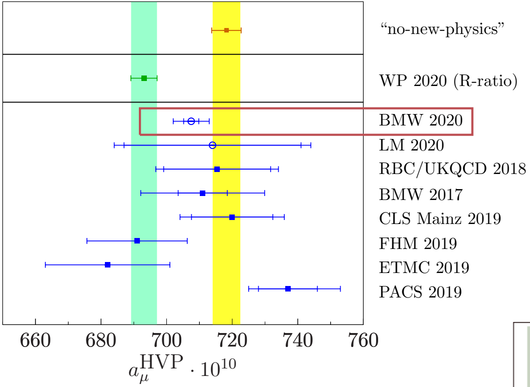
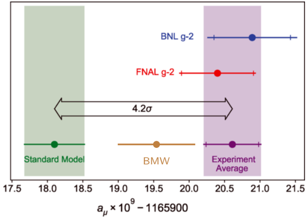


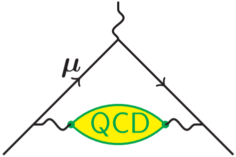
- recent lattice result by BMW

$$a_\mu^{\mathrm{HVP}}(\mathrm{BMW}) = \mathbf{707.5(5.5)} \times \mathbf{10^{-10}}$$



Hadronic Vacuum Polarisation (HVP) from the lattice

- calculate hadronic part on the lattice



- vector two-point function

$$C_{\mu\nu}(t) = \sum_{\vec{x}} \langle J_\mu(t, \vec{x}) J_\nu(0) \rangle$$

- electromagnetic current

$$J_\mu = \frac{2}{3}\bar{u}\gamma_\mu u - \frac{1}{3}\bar{d}\gamma_\mu d - \frac{1}{3}\bar{s}\gamma_\mu s \; + \ldots$$

- a_μ from $C(t)$ [T. Blum, Phys.Rev.Lett.**91**, 052001 (2003); Bernecker and Meyer, Eur.Phys.J.**A47**, 148 (2011)]

$$a_\mu^{\mathrm{HVP}} = \sum_t w_t C_{ii}(t) \qquad \text{with kernel function } w_t$$

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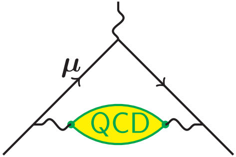

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- flavour decomposition (isospin symmetric QCD $u = d = \ell$)

$$C(t) = \frac{5}{9}C^\ell(t) + \frac{1}{9}C^s(t) + \frac{4}{9}C^c(t)$$

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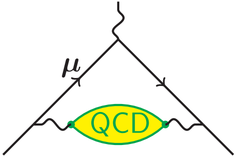

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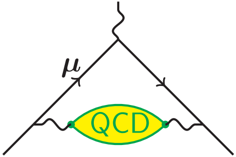

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Flavour decomposition of HVP

- flavour decomposition

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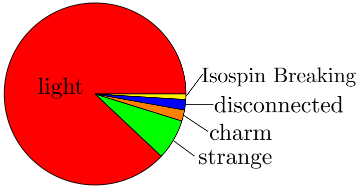
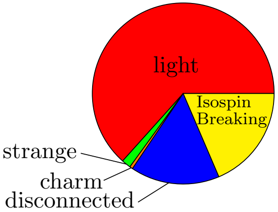
- White Paper lattice average $a_\mu^{\text{HVP}}(\text{lat}) = 711.6(18.4) \times 10^{-10}$

contributions to $a_\mu^{\text{HVP}}(\text{lat})$

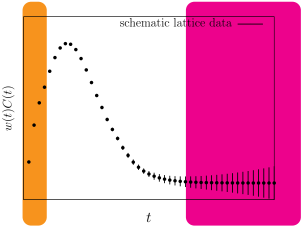


contributions to $\Delta a_\mu^{\text{HVP}}(\text{lat})$

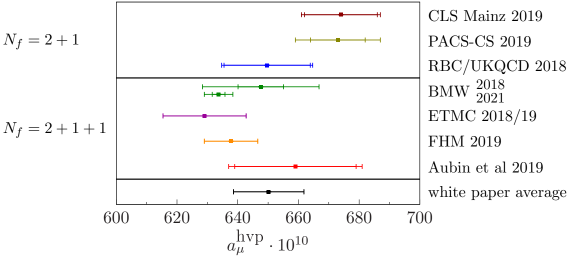


Light-quark contribution

- main challenges:
  - statistical noise at large t
  - finite volume effects (largest at large t)
  - discetisation effects at small t
- $a_\mu^{\mathrm{HVP}} = \sum_t w_t C(t)$



- summary of available lattice results



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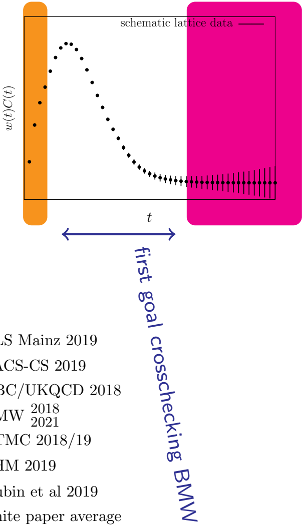

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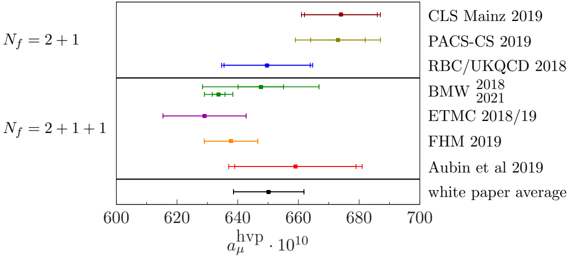


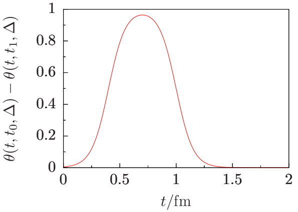
Lattice Cross Checks - Window method

- a_μ^{HVP} from intermediate window $a_\mu = a_\mu^{\mathrm{SD}} + a_\mu^{\mathrm{W}} + a_\mu^{\mathrm{LD}}$

  [T. Blum, P. Boyle, VG *et al* Phys.Rev.Lett. 121 (2018) 022003]

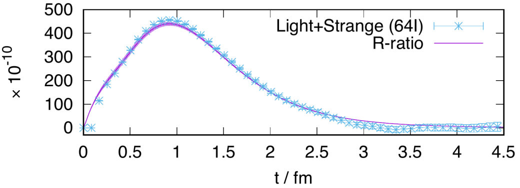
$$a_\mu^{\mathrm{W}} = \sum_t w_t C(t)[\theta(t, t_0, \Delta) - \theta(t, t_1, \Delta)]$$

e.g. $t_0 = 0.4$ fm to $t_1 = 1.0$ fm



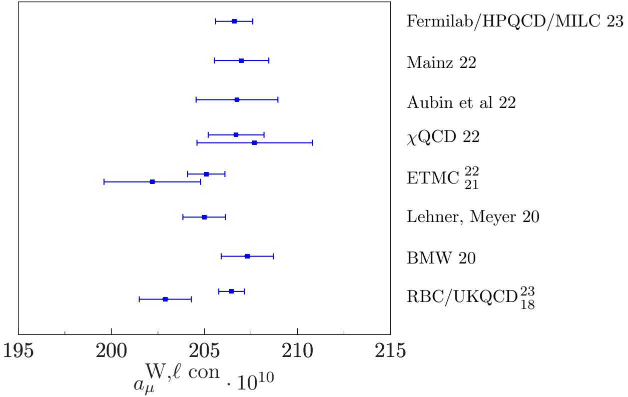
- originally proposed to combine R-ratio and lattice results

  [T. Blum, P. Boyle, VG *et al* Phys.Rev.Lett. 121 (2018) 022003]



Comparison Light-quark connected Window

- Agreement: compare "Standard" window $t_0 = 0.4$ fm to $t_1 = 1.0$ fm
- Many collaborations have calculated this recently
- good agreement between lattice calculations!



RBC/UKQCD collaboration

UC Berkeley/LBNL

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An update of Euclidean windows of the hadronic vacuum polarization

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(RBC and UKQCD Collaborations)

arXiv:2301.08696

RBC/UKQCD window result – ensembles

- Möbius Domain Wall Fermions
- three lattice spacings at the physical point with $N_f = 2 + 1$

ID	a^{-1}/GeV	$L^3 \times T \times L_s$	m_π/MeV	m_K/MeV
48I	**1.7312(28)**	$48^3 \times 96 \times 24$	**139.32(30)**	**499.44(88)**
64I	**2.3549(49)**	$64^3 \times 128 \times 12$	**138.98(43)**	**507.5(1.5)**
96I	**2.6920(67)**	$96^3 \times 192 \times 12$	**131.29(66)**	**484.5(2.3)**

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- additional "helper" ensembles at heavier masses

ID	a^{-1}/GeV	N_f	$L^3 \times T \times L_s$	m_π/MeV	m_K/MeV	m_{D_s}/GeV
1	**1.7310(35)**	2+1	$\mathbf{32^3 \times 64 \times 24}$	**208.1(1.1)**	**514.0(1.8)**	–
2	**1.7257(74)**	2+1	$\mathbf{24^3 \times 48 \times 32}$	**285.4(2.9)**	**537.8(4.6)**	–
3	**1.7306(46)**	2+1	$\mathbf{32^3 \times 64 \times 24}$	**211.3(2.3)**	**603.8(6.1)**	–
4	**1.7400(73)**	2+1	$\mathbf{24^3 \times 48 \times 24}$	**274.8(2.5)**	**530.1(3.1)**	–
5	**1.7498(73)**	2+1+1	$\mathbf{24^3 \times 48 \times 24}$	**279.8(3.5)**	**539.1(5.3)**	**1.9902(69)**
7	**1.7566(81)**	2+1+1	$\mathbf{24^3 \times 48 \times 24}$	**272.5(5.9)**	**523(10)**	**1.3882(57)**
A	**1.7556(83)**	2+1	$\mathbf{24^3 \times 48 \times 8}$	**307.4(3.5)**	**557.3(5.7)**	–

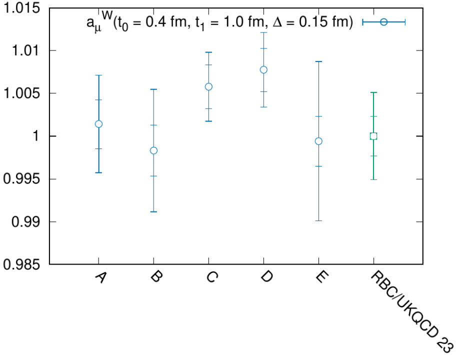
RBC/UKQCD window result – blinding

- blinded analysis, to avoid bias towards other results
- five different analysis groups
- vector two-point function blinded

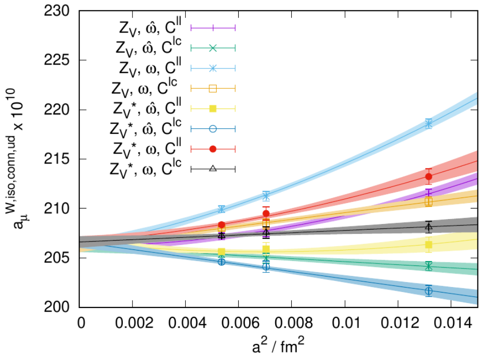
$$C_b(t) = (b_0 + b_1 a^2 + b_2 a^4) C_0(t)$$

with random coefficients b_0, b_1 and b_2 different for each group

- relative unblinding



RBC/UKQCD window result – continuum extrapolation



- isospin symmetric light-quark connected window

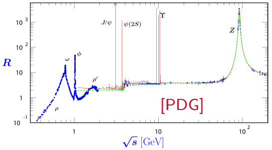
$$a_\mu^{\text{W, iso, conn, ud}} = \mathbf{206.36(44)(43)} \times \mathbf{10^{-10}}$$

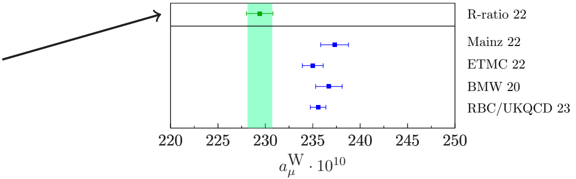
- total window (using RBC/UKQCD 2018 results for other flavours)

$$a_\mu^{\text{W}} = \mathbf{235.56(65)(50)} \times \mathbf{10^{-10}}$$

Window comparison with R-ratio

- compare ***R***-ratio with lattice using window quantity





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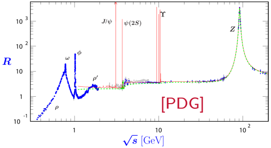
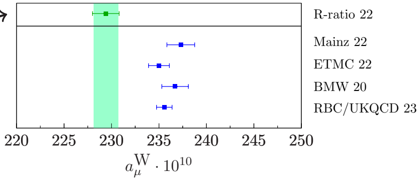
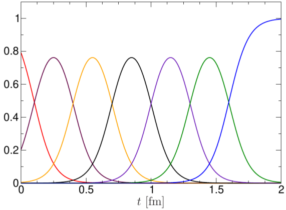
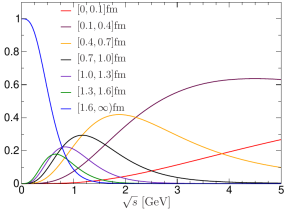




- scrutinise (potential) tensions between ***R***-ratio and lattice by considering different windows [G. Colangelo *et al.*, Phys.Lett.B 833 (2022) 137313]





- see also [D. Bioto et al, Phys.Rev.D 107 (2023) 3, 034512; C. Alexandrou et al, arXiv:2212.08467]

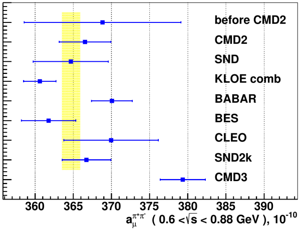
Quo Vadis g-2?

- "***R***-ratio Scenario": lattice consistent with ***R***-ratio (unlikely?)
- "BMW Scenario": Other (full) lattice calculations agree with BMW
  $\longrightarrow$ tension between lattice and ***R***-ratio?

Quo Vadis g-2?

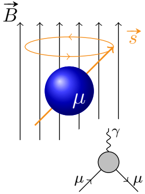
- "$\boldsymbol{R}$-ratio Scenario": lattice consistent with $\boldsymbol{R}$-ratio (unlikely?)
- "BMW Scenario": Other (full) lattice calculations agree with BMW $\rightarrow$ tension between lattice and $\boldsymbol{R}$-ratio?

- New results for $e^+e^- \rightarrow \pi^+\pi^-(\gamma)$ from CMD-3 disagree with KLOE and BABAR [CMD-3, arXiv:2302.08834]



- origin of tension not yet understood
- CMD3 would bring R-ratio closer to "no-new-physics" (and BMW)

Summary



- Muon $g-2$ promising quantity for finding new physics
  $\rightarrow$ currently: $\mathbf{4.2\sigma}$ tension between experiment and theory
- Theory error dominated by Hadronic Vacuum Polarisation
- recent lattice calculation closer to experiment, in tension with R-ratio
  $\rightarrow$ further precise lattice calculations (of full HVP) needed

- Are we about to find new physics with a_μ?

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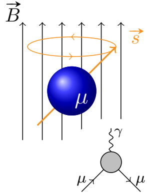

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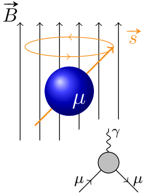

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Thank you!