Nested Sampling for Lattice Field Theory

David Yallup (dy297@cam.ac.uk)

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UKLFT meeting

Live slides with animation available:
[yallup.github.io/uklft]

Nested Sampling

Technique for computation of multidimensional integrals [Ashton et al - Nature review paper]

> Widespread adoption in Bayesian evidence integrals

$$P(\theta|X) = \frac{\mathcal{L}(X|\theta) \times \Pi(\theta)}{Z}$$

$$Z = \int D\theta \mathcal{L}(X|\theta)\Pi(\theta)$$

Cambridge Cosmology (KICC and Cavendish AP) one of the driving forces in development:

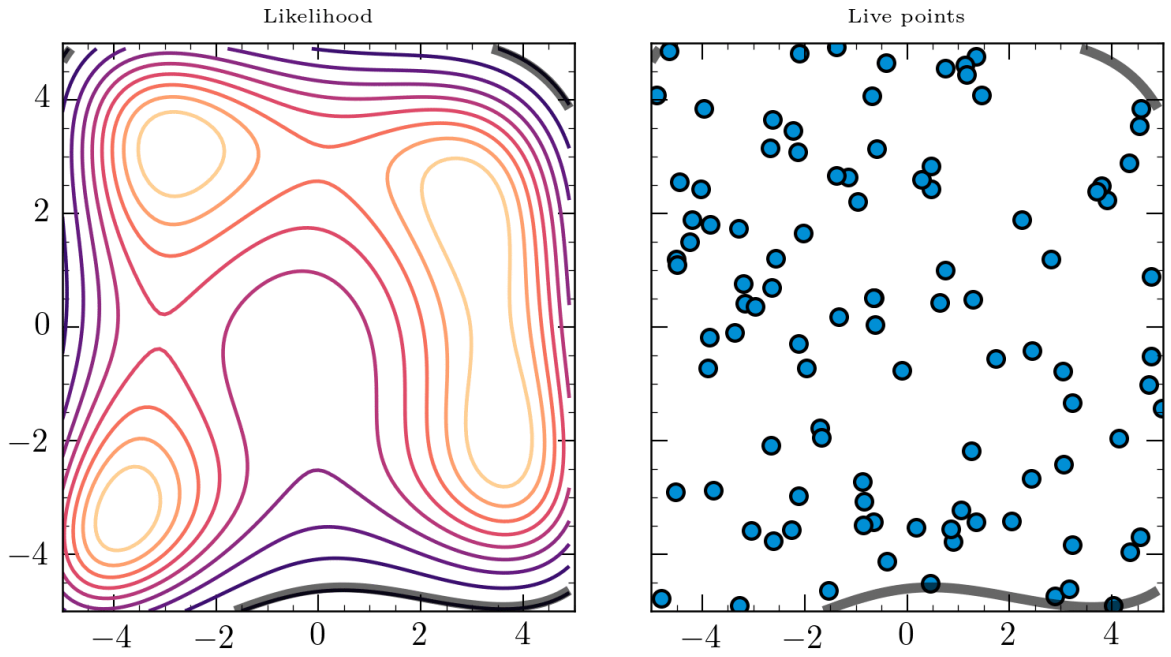
+ DY (Inference and ML applications)
+ Will Barker (Gravity on the lattice)
+ Will Handley (all of the above)
+ Boris Deletic (Part III student, gradients in NS)

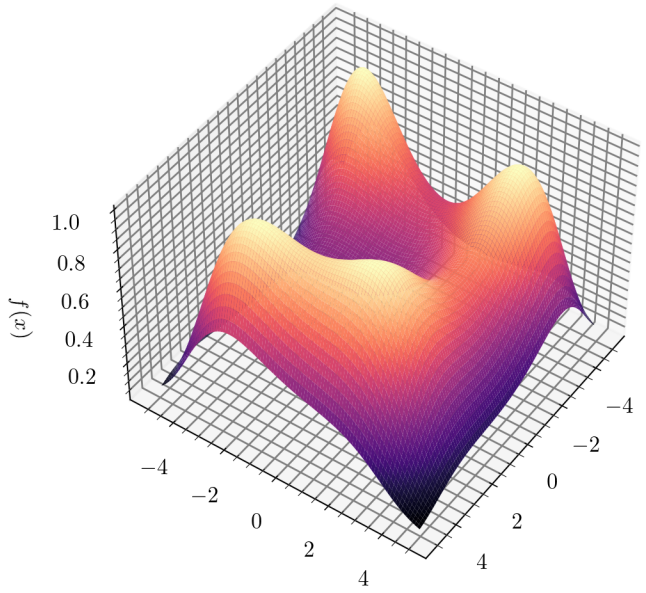
What does this bring to the table

Rather than a ensemble of chains, construct a chain of ensembles

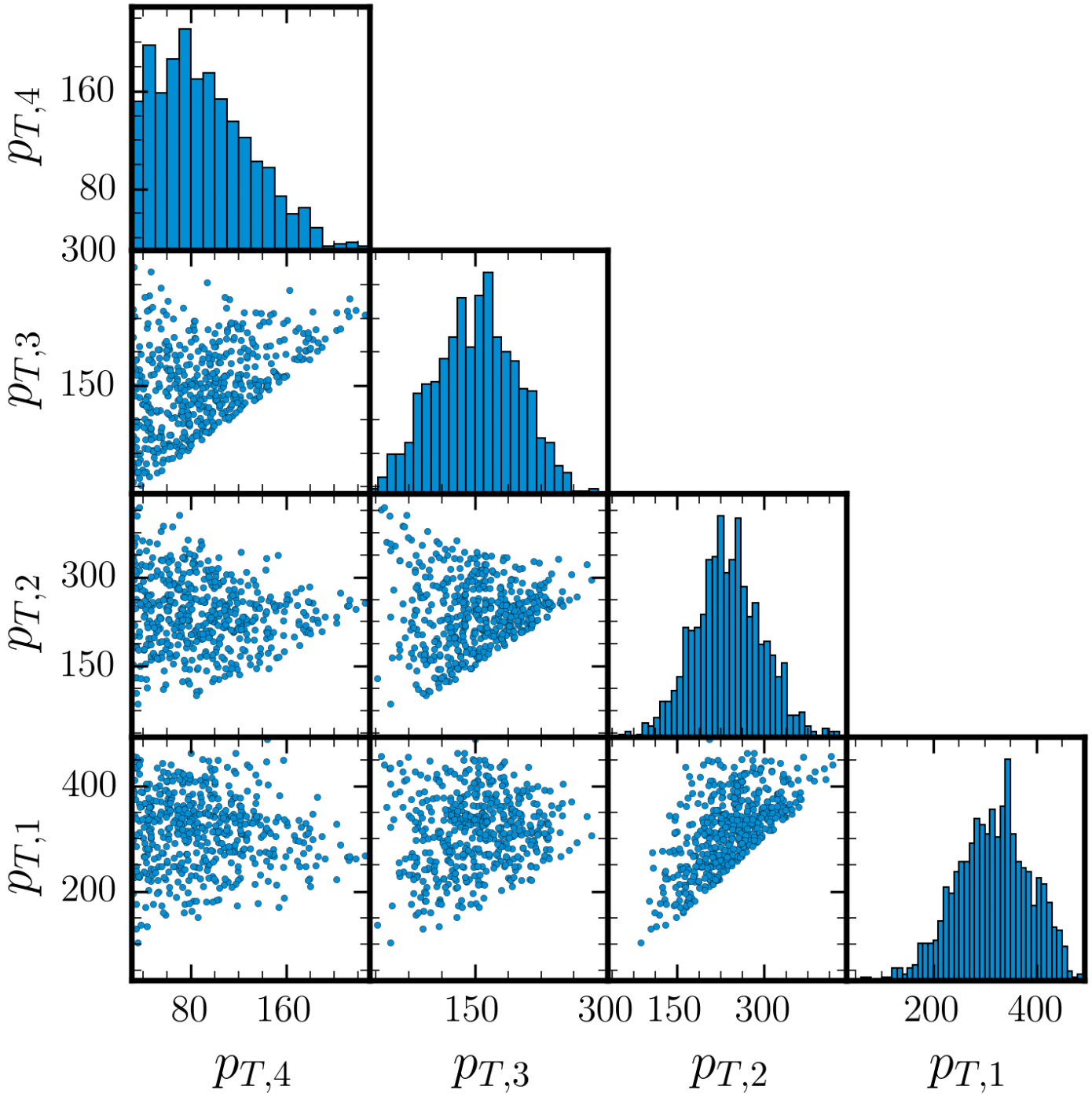
[Original paper - Skilling]

[Our implementation, PolyChord - Handley et al]





$$f(x) \propto -\exp[(x^2 + y - 11)^2 + (x + y^2 - 7)^2]$$



Non Bayesian inference example

Matrix elements @ LHC

Pose phase space integration as [2205.02030]

$$\sigma = \int_{\Omega} d\Phi |\mathcal{M}|^2(\Phi)$$

$$\mathcal{Z} = \int d\theta \mathcal{L}(\theta)\Pi(\theta)$$

Explore scale choice as model comparison problem

nb: ME as Likelihood, not fitting to data

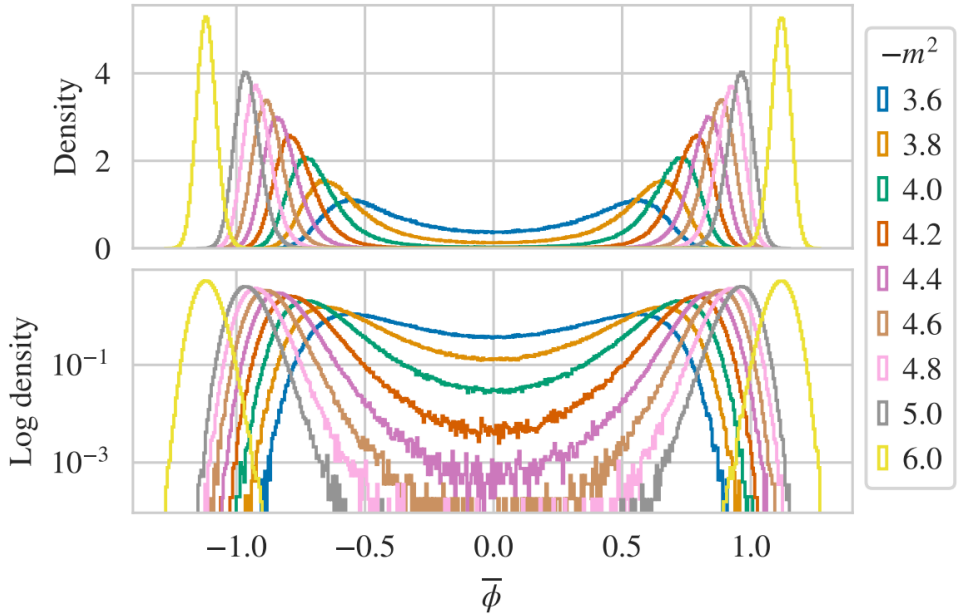
Lattices and Machine Learning

Interest in ML methods to overcome/complement difficulties with HMC

"Flow-based sampling for multimodal distributions in lattice field theory" [2107.00734]

$$S_E(\phi) = \sum_x (\sum_{\mu=1}^{D} \frac{1}{2}(\phi(x+\hat{\mu}) - \phi(x))^2 + \frac{1}{2}m^2\phi(x)^2 + \lambda\phi(x)^4)$$

Sample on 2D, 10×10 lattice



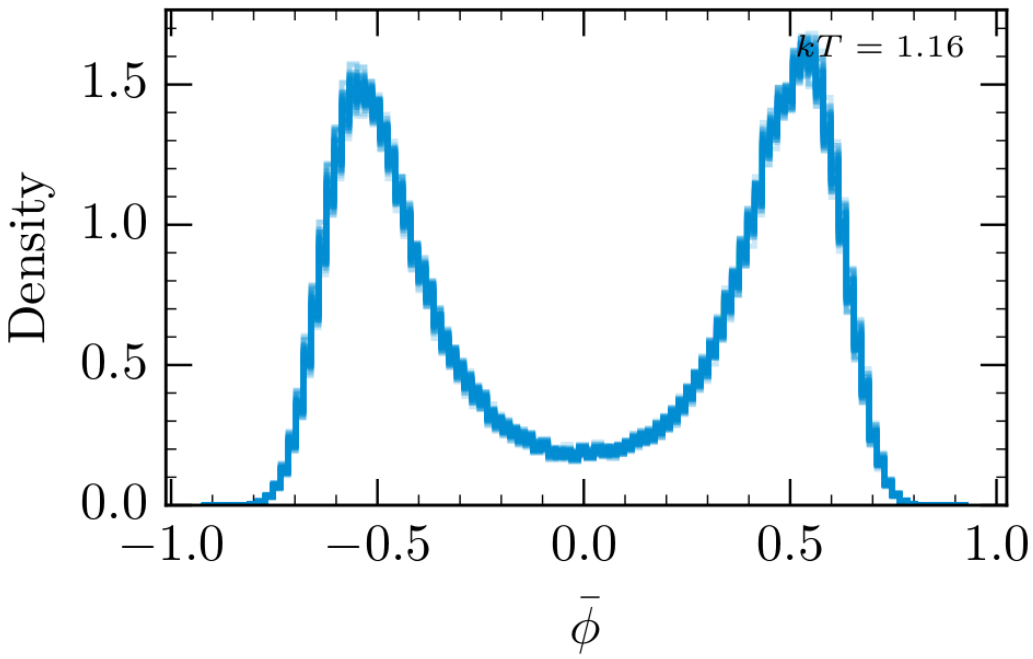
NFs learn mapping of prior $\longrightarrow$ posterior, Nested sampling compresses prior $\longrightarrow$ posterior.
Roughly shared dimension limitations, shared promises...

Nested sampling simple lattices

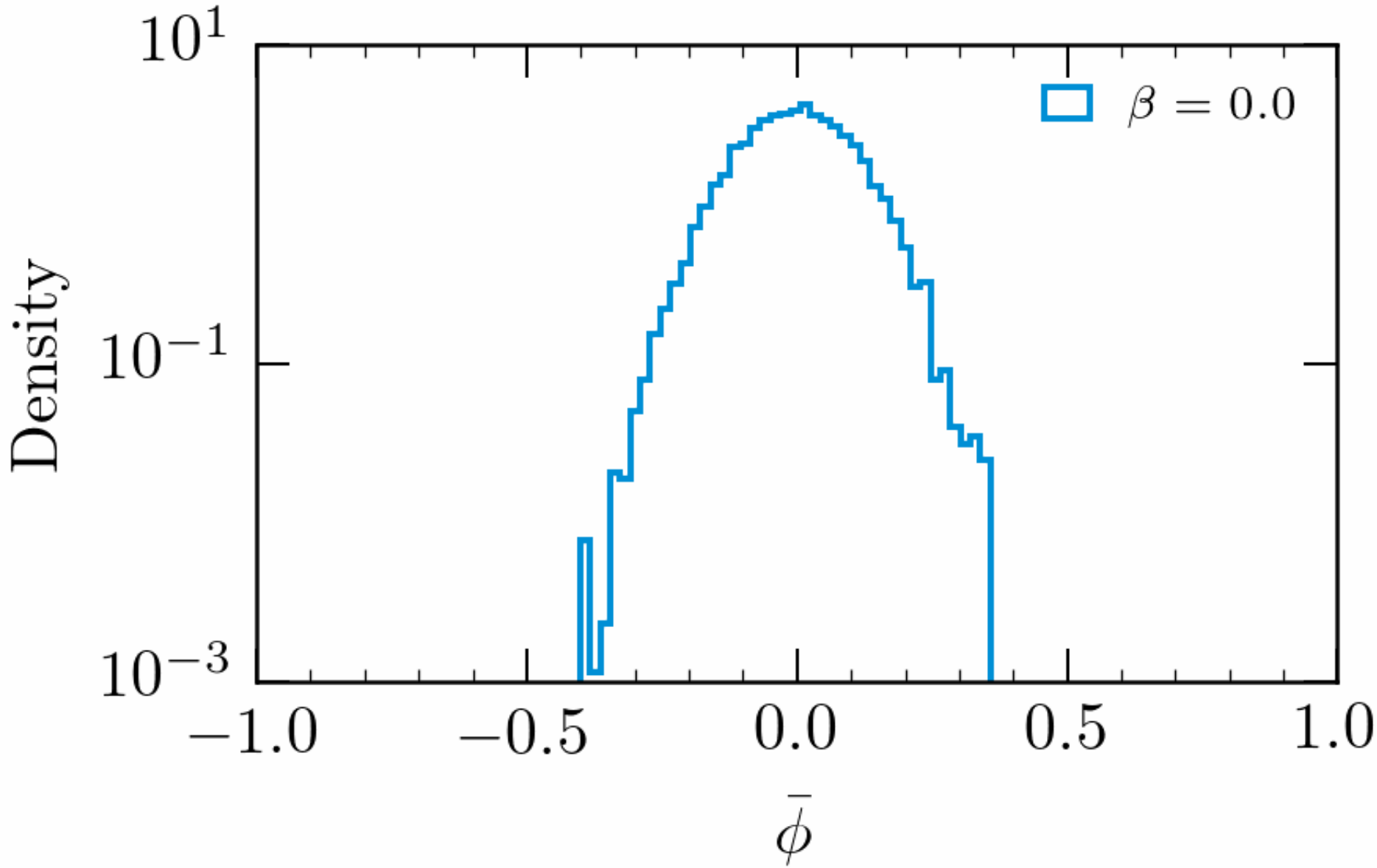
Same scalar ϕ^4 theory on 10×10 lattice as previous slide. NS computes partition function,

$$Z(\beta) = \int D\phi \, e^{-\beta S(\phi)}, \quad \beta = \frac{1}{kT}$$

- The density of states (prior volume estimation) is the missing piece in inference, normally avoided/cancelled in traditional methods.
- The sampling process is athermal, and invariant under monotonic transformations of the sampled distribution
- Clustering can be inserted at runtime



By appropriate re-weighting, we can post-process the posterior samples to be at any temperature.


10^1
10^{-1}
10^{-3}
Density
$\beta = 0.0$
-1.0
-0.5
0.0
0.5
1.0
$\bar{\phi}$