Quantum computing for lattice supersymmetry

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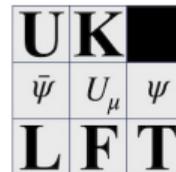
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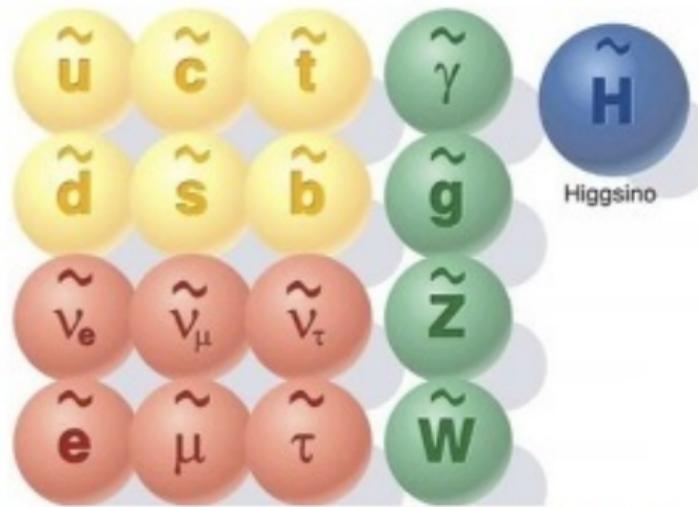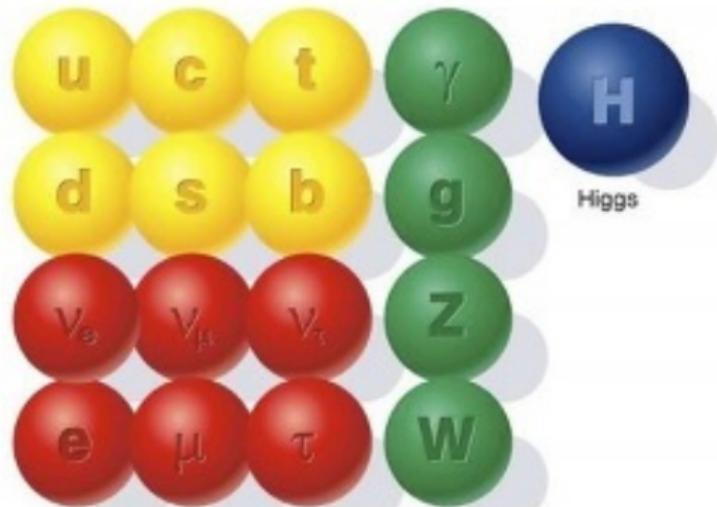
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Outline

Motivation

Physics Motivation

Supersymmetry as an extension to the standard model

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Dynamical symmetry breaking
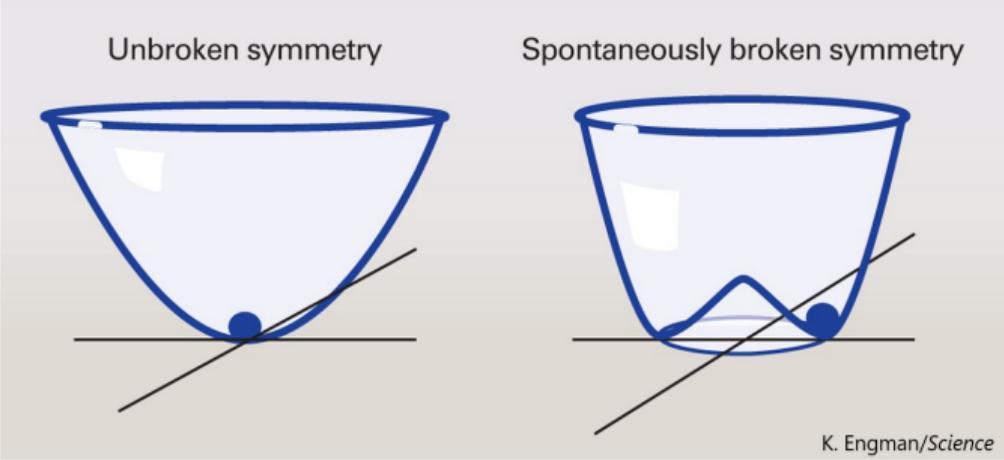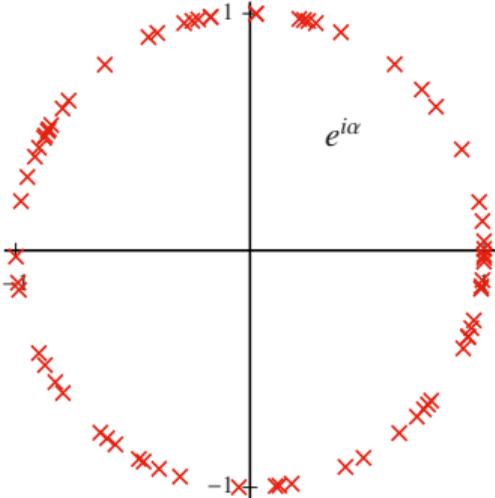
Holographic duality
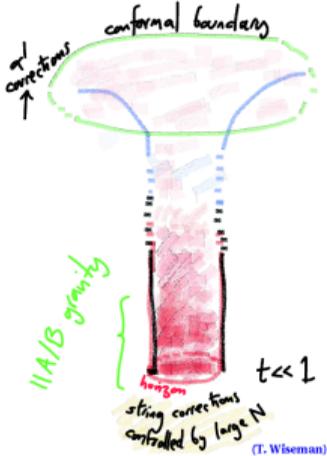
Sign problem

Lattice $\mathcal{N} = 4$ SYM

$e^{i\alpha}$

NISQ devices
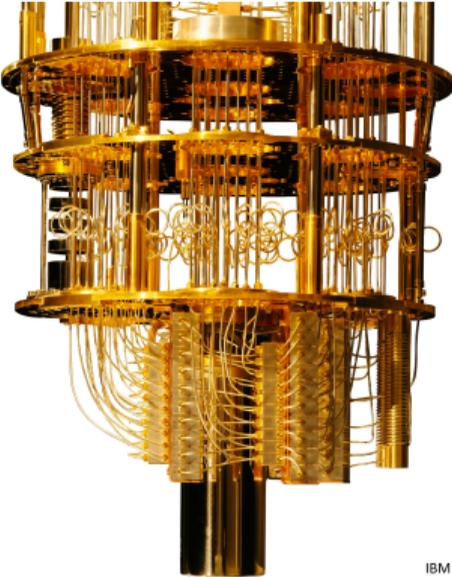
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Introduction

Supersymmetry Facts

Hamiltonian

$$H = Q^2$$

- $E \geq 0$
- Supersymmetry conserved if $\langle \Omega | H | \Omega \rangle = 0$ [Witten, Nucl. Phys. B 188,513 (1981)]
- Non-zero E states appear in pairs

- Lagrangian formalism

Path integral

$$\langle \mathcal{O} \rangle = \frac{1}{Z} \int d\hat{q} \, d\hat{b} \, d\hat{b}^\dagger \, \mathcal{O} \, e^{iS(\hat{q}, \hat{b}, \hat{b}^\dagger)}$$

- Condition for symmetry breaking

Witten index

$$\mathcal{W} = \mathrm{Tr} \left[(-1)^F e^{-iHt} \right] = 0$$

Quantum computing
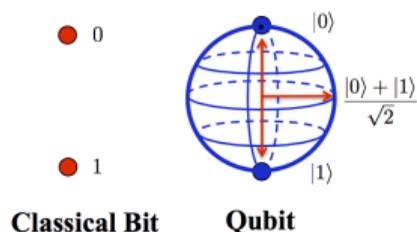
- Map d.o.f of bosons/fermions to qubit d.o.f.

- Choose quantum computation

  ▸ VQE - variational method to solve for lowest lying eigenvalues

  ▸ Time evolution - study dynamics of Hamiltonian

Mapping degrees of freedom

- Bosonic
  - Harmonic oscillator basis with cutoff Λ excitations
  - n excitation to binary string of length N^q, $|n\rangle \rightarrow \left| \sum_i^{N^q-1} a_i 2^i \right\rangle$

Matrix Elements

$|0\rangle \langle 1| = (X + iY)/2, \quad |1\rangle \langle 0| = (X + iY)/2,$
$|0\rangle \langle 0| = (1 + Z)/2, \quad |1\rangle \langle 1| = (1 - Z)/2$

- Fermionic

Jordan-Wigner Transformation

$\hat{b}^\dagger = \frac{1}{2}(X - iY), \ \hat{b} = \frac{1}{2}(X + iY)$

This talk

Two supersymmetric systems

- Classical diagonalization of H

- VQE estimate of ground state energy

- CX gate counts for real time evolution

$0+1D$ supersymmetric quantum mechanics

$0 + 1$ dimensional quantum mechanics

Hamiltonian

$$H = \frac{1}{2}\left(\hat{p}^2 + [W'(\hat{q})]^2 - W''(\hat{q})\left[\hat{b}^\dagger, \hat{b}\right]\right)$$

- Superpotentials W studied with complex Langevin [Joseph-Kumar, arxiv:2011.08107]:

  - HO: $\frac{1}{2}m\hat{q}^2$ preserves supersymmetry

  - AHO: $\frac{1}{2}m\hat{q}^2 + \frac{1}{4}g\hat{q}^4$ preserves supersymmetry

  - DW: $\frac{1}{2}m\hat{q}^2 + g(\frac{1}{3}\hat{q}^3 + \hat{q}\mu^2)$ breaks supersymmetry

- For HO with $\Lambda=2$, $H = 1.5 I^0 I^1 + I^0 Z^1 - 0.5 Z^0 I^1$

0+1 harmonic oscillator spectrum
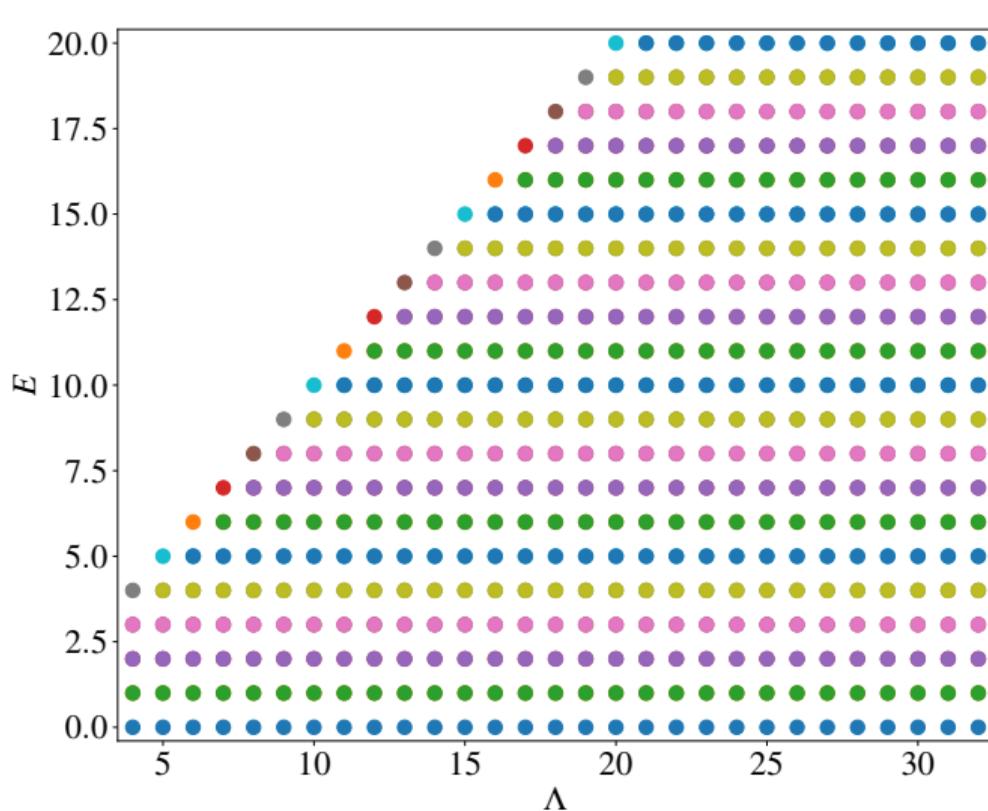
$m = 1.0$

Supersymmetric anharmonic oscillator spectrum
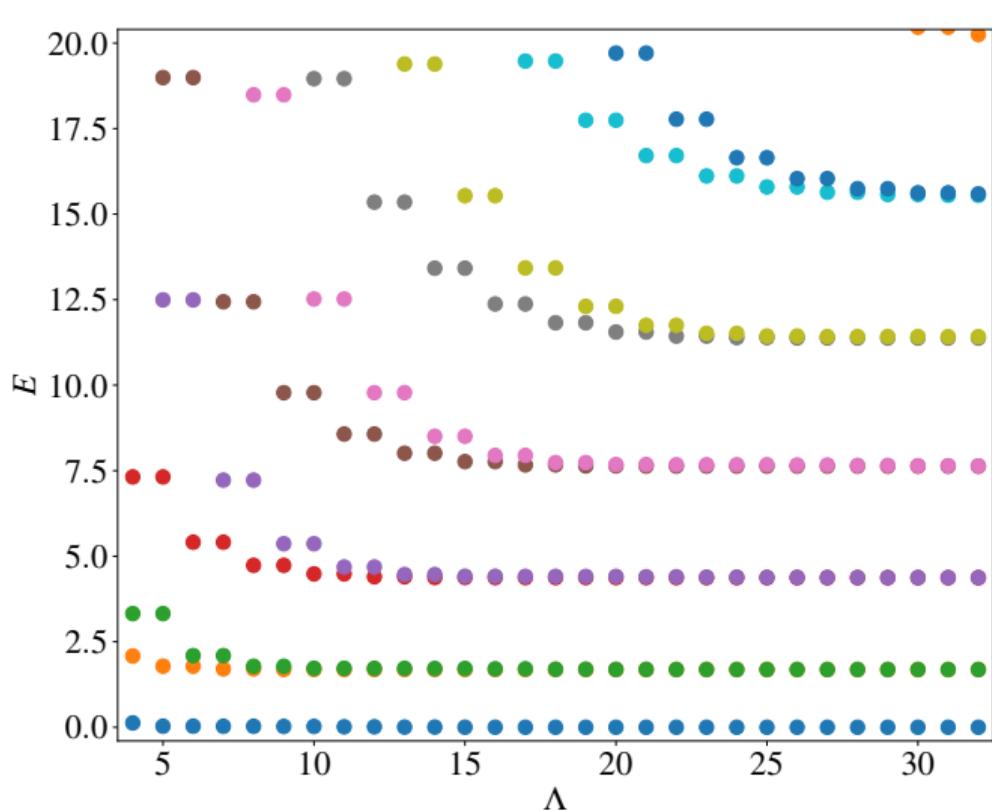
$m = 1, g = 1$

Supersymmetric anharmonic oscillator spectrum

$m = 1,\ g = 1$

Double well spectrum

$m = 1,\ g = 1,\ \mu = 1$

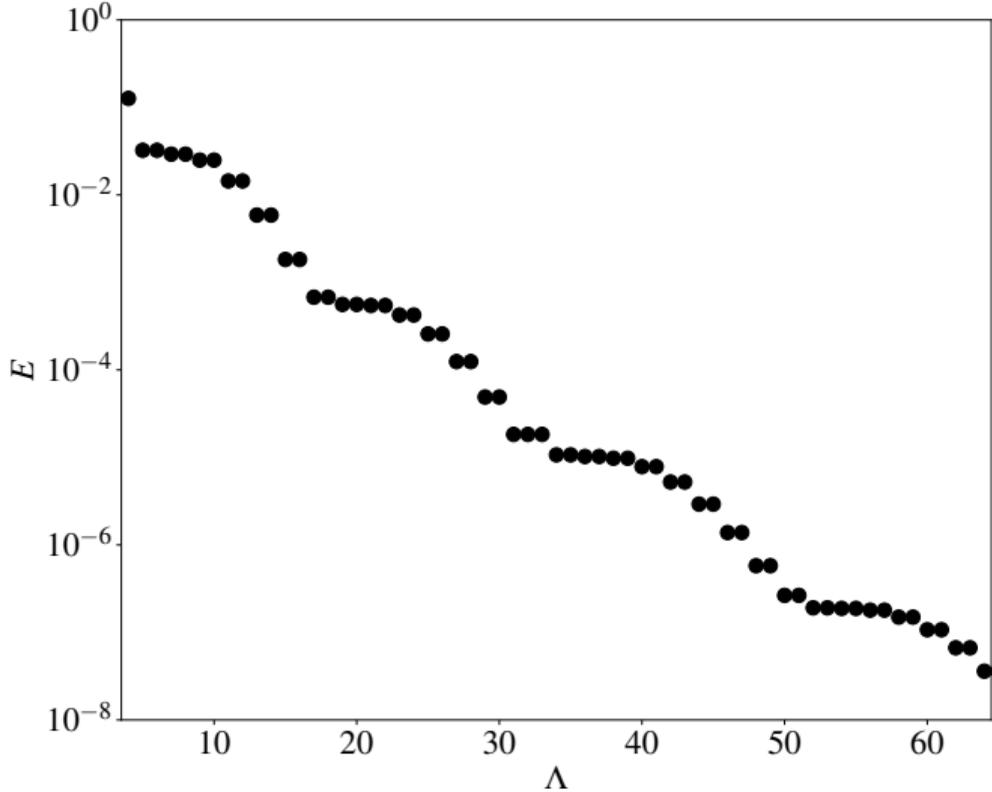| Λ	Exact	VQE
2	0.00e+00	5.34e-10
4	0.00e+00	1.07e-09
8	0.00e+00	4.06e-09
16	0.00e+00	1.13e-08
32	0.00e+00	4.81e-08

(a) Harmonic oscillator.

Λ	Exact	VQE
2	9.38e-01	9.38e-01
4	1.27e-01	1.27e-01
8	2.93e-02	2.93e-02
16	1.83e-03	6.02e-02
32	1.83e-05	6.63e-01

(b) Anharmonic oscillator.

Λ	Exact	VQE
2	1.08e+00	1.08e+00
4	9.15e-01	9.15e-01
8	8.93e-01	8.93e-01
16	8.92e-01	8.94e-01
32	8.92e-01	8.95e-01

(c) Double well.

VQE

Λ	Exact	VQE
2	0.00e+00	5.34e-10
4	0.00e+00	1.07e-09
8	0.00e+00	4.06e-09
16	0.00e+00	1.13e-08
32	0.00e+00	4.81e-08

(a) Harmonic oscillator.

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(b) Anharmonic oscillator.

Λ	Exact	VQE
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32	8.92e-01	8.95e-01

(c) Double well.

- Similar problems to (b) in BMN matrix model [Rinaldi et al, PRX Quantum 3 (2022)]

Trotter step costs

Single Trotter Step
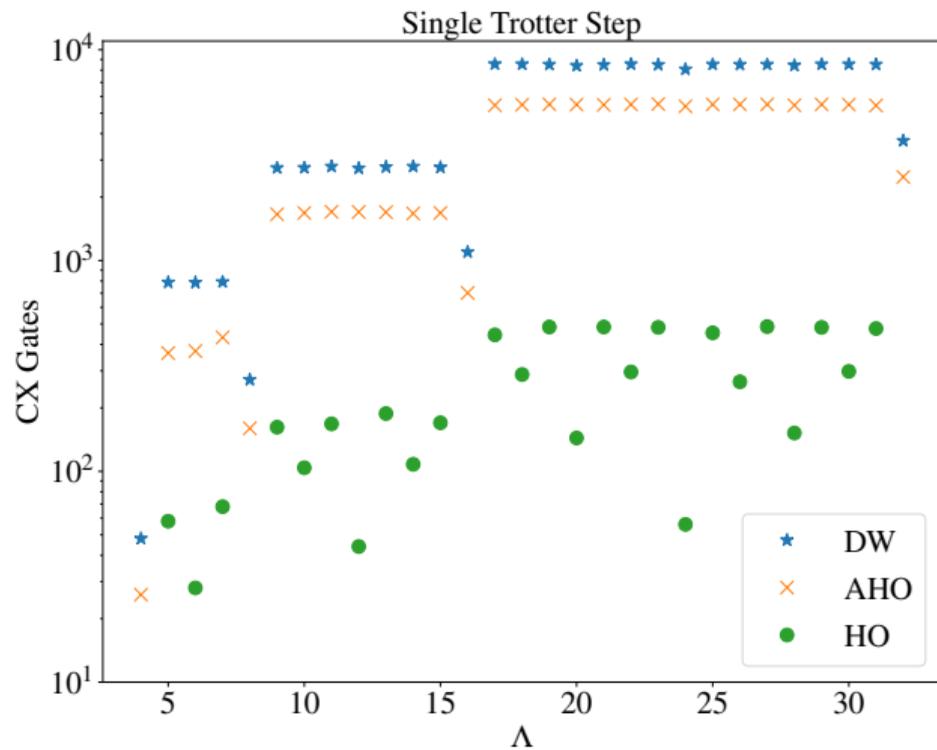
$1 + 1D$ Wess-Zumino model

1 + 1D Wess-Zumino model

Lattice Hamiltonian

$$H = \sum_{n=0}^{N-1} \left[\frac{p_n^2}{2a} + \frac{a}{2} \left(\frac{\phi_{n+1} - \phi_{n-1}}{2a} \right)^2 + \frac{a}{2} V(\phi_n)^2 + a V(\phi_n) \frac{\phi_{n+1} - \phi_{n-1}}{2a} \right.$$

$$\left. + (-1)^n V'(\phi_n) \left(\chi_n^\dagger \chi_n - \frac{1}{2} \right) + \frac{1}{2a} \left(\chi_n^\dagger \chi_{n+1} + \chi_{n+1}^\dagger \chi_n \right) \right], \qquad (1)$$
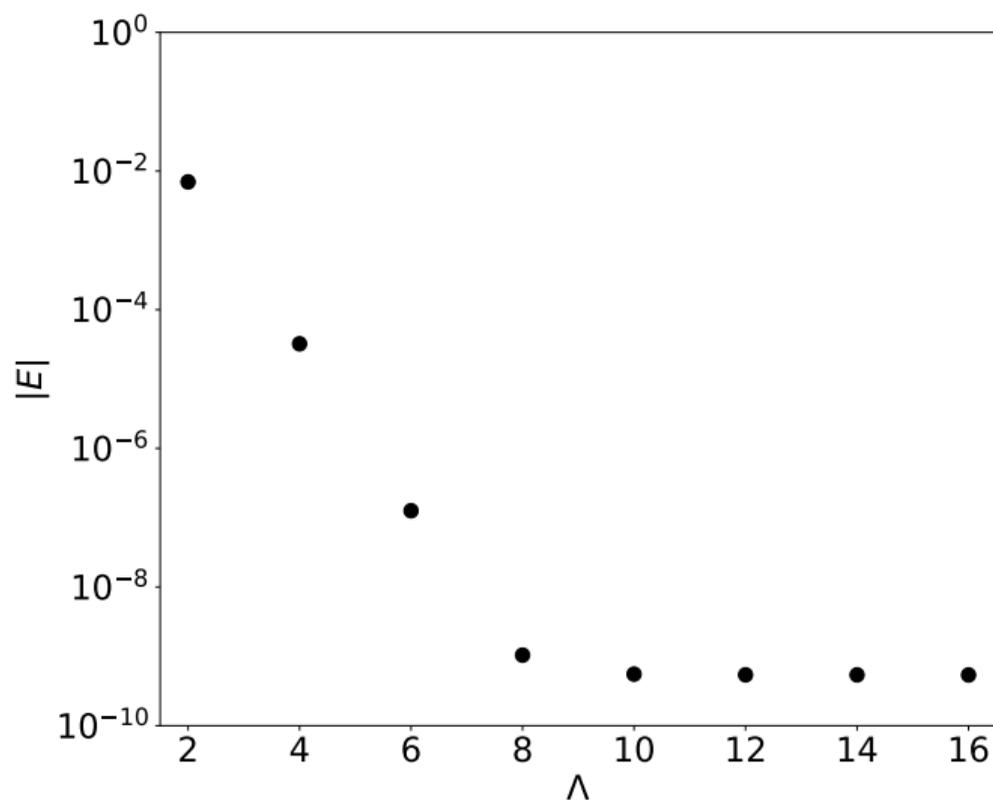
- Potentials
  $$\dim(H) = \Lambda^N \times 2^N$$
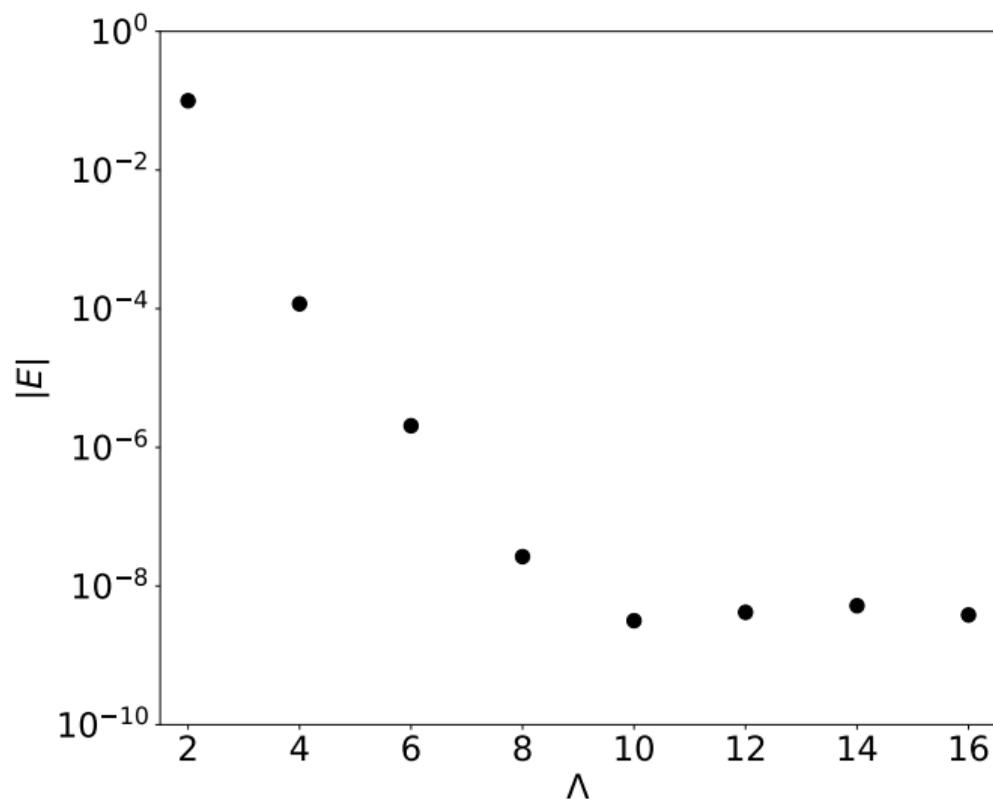 - ▶ Linear: $m\phi$

  - ▶ Quadratic: $c + \phi^2$ [Beccaria, Campostrini, Feo, hep-lat/0109005 (2001)]
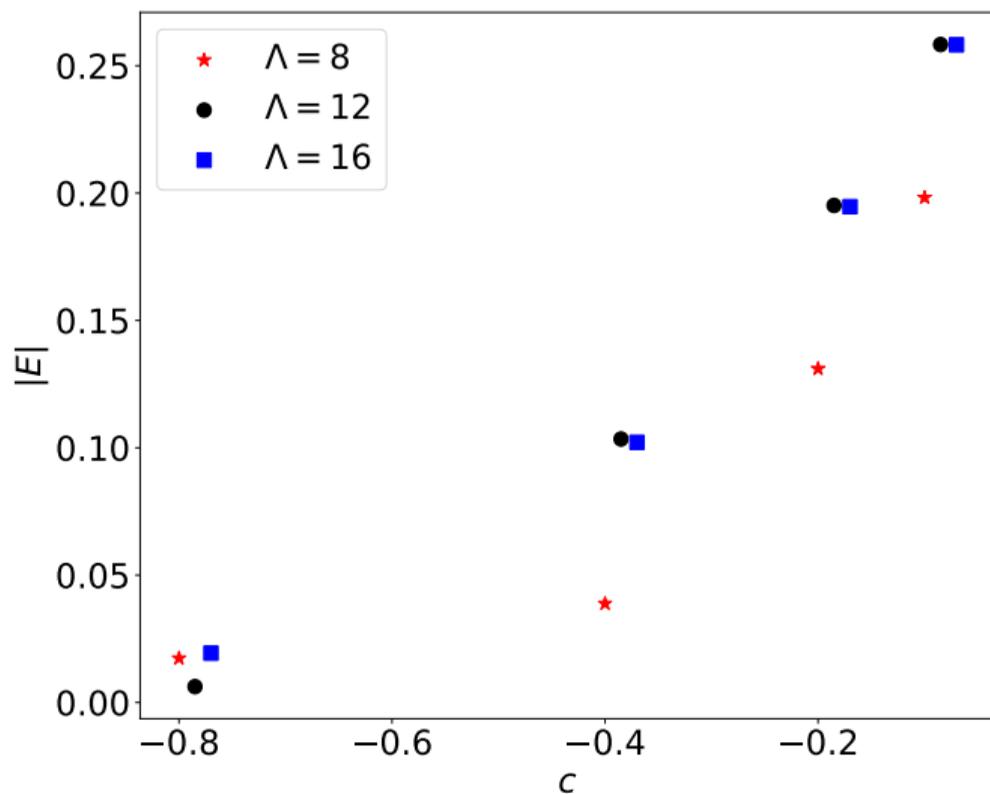
$m = 1$

3 site linear potential

$m = 1$

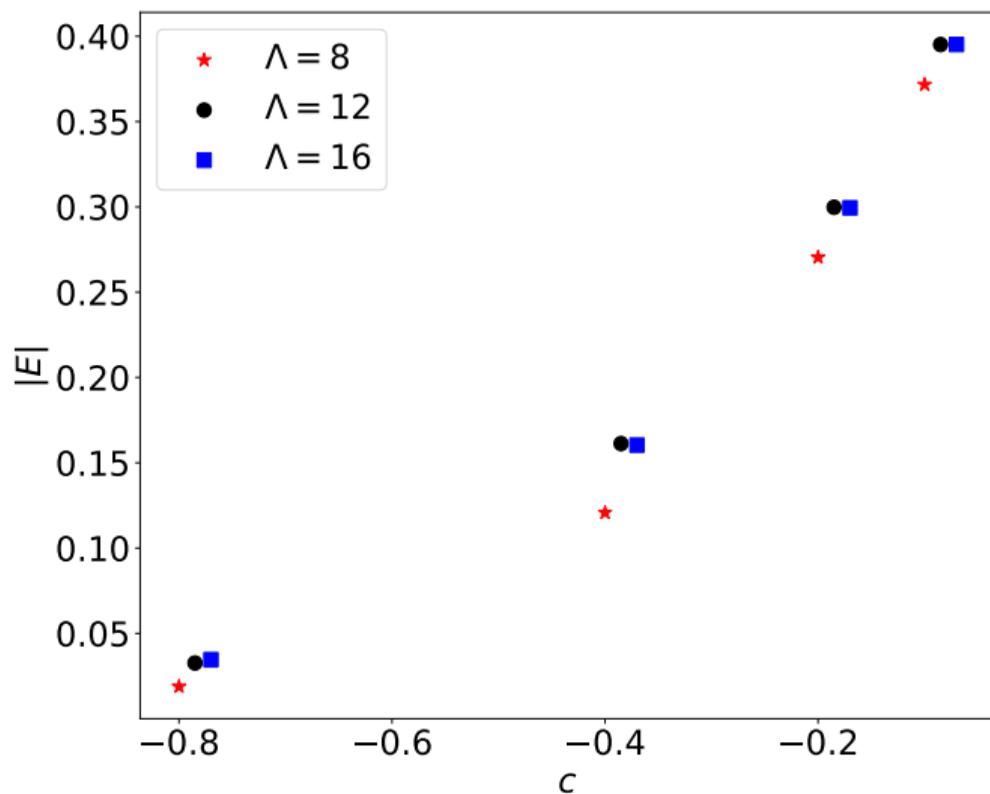# 2 site quadratic potential

$m = 1$

3 site quadratic potential

$m = 1$

VQE

N	Λ	Exact	VQE
2	2	6.97e-03	6.97e-03
-	4	3.22e-05	6.61e-05
-	8	1.04e-09	1.08e-01
3	2	-9.97e-02	-1.28e+00
-	4	1.17e-04	4.99e-01

(a) Linear prepotential

N	Λ	Exact	VQE
2	2	-4.87e-01	-9.11e-01
-	4	1.82e-01	2.26e-01
-	8	1.31e-01	7.49e-01
3	2	-1.98e-01	-1.28e+00
-	4	3.02e-01	5.08e-01

(b) $c = -0.2$

N	Λ	Exact	VQE
2	2	-4.87e-01	-9.11e-01
-	4	1.28e-01	-1.15e+00
-	8	-1.74e-02	6.89e-01
3	2	-1.98e-01	-1.28e+00
-	4	2.47e-01	-1.10e+00

(c) $c = -0.8$

Trotter step costs

N	Λ	CX Gates
2	2	8
-	4	252
-	8	2556
3	2	18
-	4	5728

(a) Linear prepotential

N	Λ	CX Gates
2	2	14
-	4	754
-	8	7822
3	2	30
-	4	2788

(b) $c = -0.2$

N	Λ	CX Gates
2	2	14
-	4	718
-	8	7858
3	2	30
-	4	2730

(c) $c = -0.8$

Conclusion

Conclusion

Results

- $0 + 1D$
  - Classical diagonalization works as expected
  - VQE struggles with AHO
  - Trotter step gate counts appropriate for NISQ
- $1 + 1D$
  - Classical preparation is severly limited
  - Bosonic cutoff introduces more error

Ongoing

- VQE on larger values of cutoff
- Study observables of interest as function of time
- Machine learning approach to ground state energy