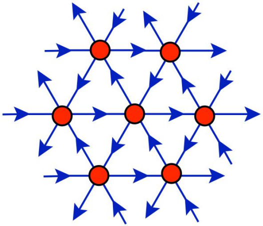
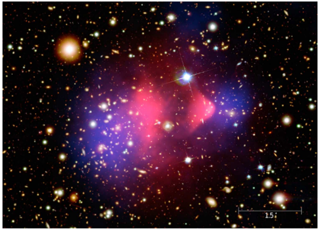
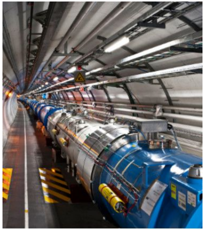
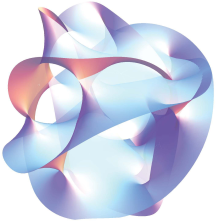
Broader applications of lattice field theory

David Schaich (University of Liverpool)

UKLFT kick-off meeting, 24 March 2021

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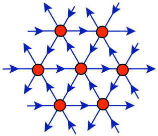
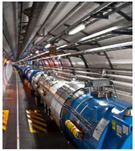
Overview and plan

Lattice field theory is a broadly applicable tool
to study strongly coupled quantum field theories

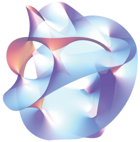


Composite dark matter and gravitational waves [2006.16429]

Composite Higgs and near-conformal dynamics [2007.01810]



Supersymmetry and holographic duality [2010.00026]



These slides: davidschaich.net/talks/2103UKLFT.pdf

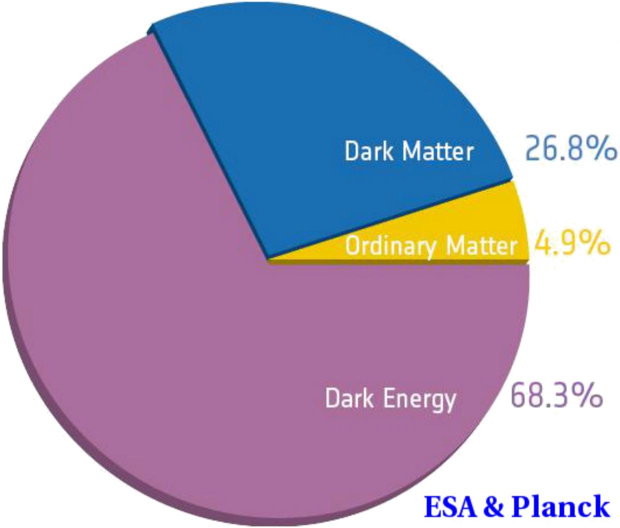
Interaction encouraged — complete coverage unnecessary

Application: Dark matter

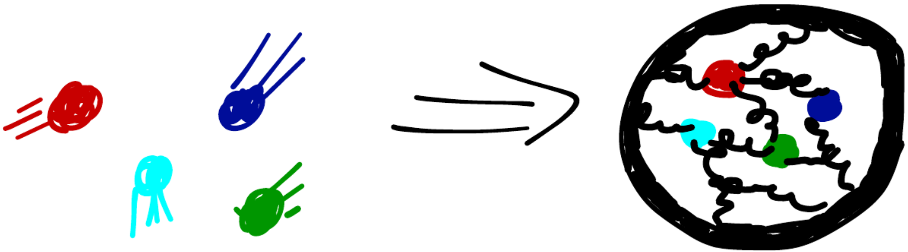
Consistent gravitational evidence from kiloparsec to Gpc scales

$$\frac{\Omega_{\text{dark}}}{\Omega_{\text{ordinary}}} \approx 5 \qquad \ldots \text{not } \; 10^5 \; \text{ or } \; 10^{-5}$$

$\longrightarrow$ non-gravitational interactions with standard model



Composite dark matter



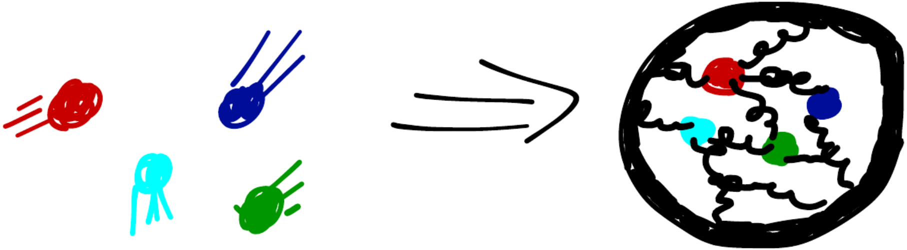
Early universe

Deconfined charged fermions $\longrightarrow$ non-gravitational interactions

Present day

Confined neutral ‘dark baryons’ $\longrightarrow$ no experimental detections so far

Composite dark matter



Experimental signals

Direct detection and collider searches depend on dark baryon **form factors**

Gravitational waves depend on dark sector **phase transitions**

Need lattice calculations for quantitative predictions

Lattice Strong Dynamics Collaboration

Argonne	Xiao-Yong Jin, James Osborn
Bern	Andy Gasbarro
Boston	Casey Berger, Rich Brower, Evan Owen, Claudio Rebbi
Colorado	Anna Hasenfratz, Ethan Neil, Curtis Peterson
UC Davis	Joseph Kiskis
Livermore	Dean Howarth, Pavlos Vranas
Liverpool	Chris Culver, DS
Michigan	Enrico Rinaldi
Nvidia	Evan Weinberg
Oregon	Graham Kribs
Siegen	Oliver Witzel
Trieste	James Ingoldby
Yale	Thomas Appelquist, Kimmy Cushman, George Fleming

Exploring the range of possible phenomena in strongly coupled field theories

Direct detection and collider searches

[PRL **115** 171803; PRD **92** 075030]

Lattice studies of four-flavour SU(4) dark sector

$\longrightarrow$ lightest **scalar** ‘baryon’ is stable dark matter candidate

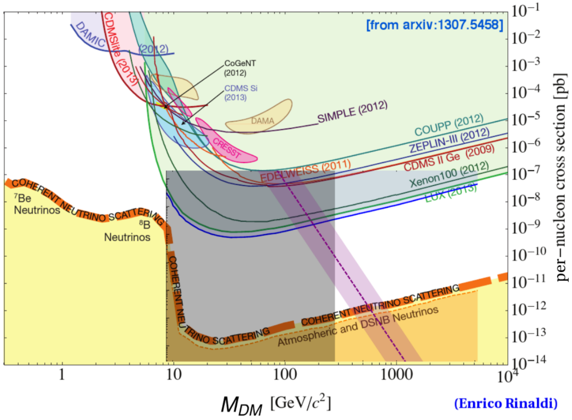
Direct detection

Symmetries

$\longrightarrow$ electric **polarizability** is leading interaction

Collider searches

Charged ‘meson’ Drell–Yan rules out shaded region



Gravitational waves

arXiv:2006.16429

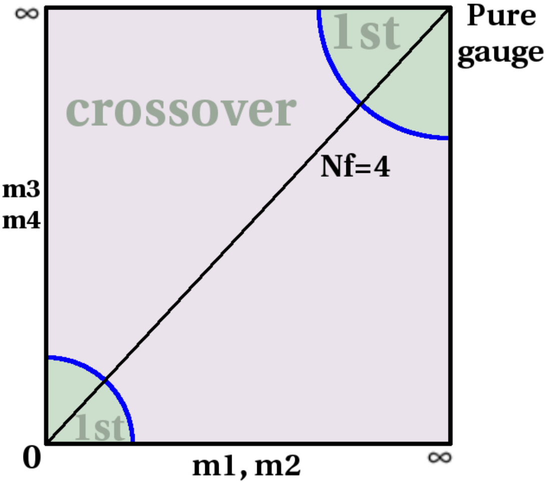
First-order confinement transition $\longrightarrow$ stochastic background of grav. waves

Pure-gauge transition is first order

Becomes stronger as N increases

First-order transition persists
for sufficiently heavy fermions

Four-flavour SU(4) lattice studies
$\longrightarrow$ need $M_P/M_V \gtrsim 0.9$

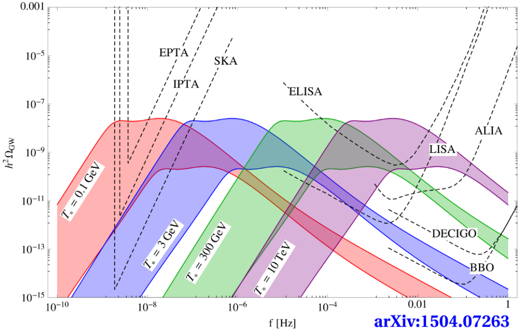


Promising directions for composite dark matter

First-order transition located $\longrightarrow$ now need lattice analyses of its properties

Currently computing **latent heat**

Gravitational wave spectrum
also sensitive to supercooling,
bubble nucleation rate & wall speed



arXiv:1504.07263

Another investigation currently underway

Baryon–baryon scattering to explore dark 'nuclei' and sub-galactic structure

Application: Composite Higgs

Large Hadron Collider priority

Study fundamental nature of the Higgs

Composite Higgs sector
can stabilize electroweak scale

New strong dynamics must differ from QCD
$\longrightarrow$ need lattice calculations

Near-conformality in composite Higgs models

Nearly conformal dynamics makes scale separation natural

$\longrightarrow$ consistent with non-observation of new particles at LHC

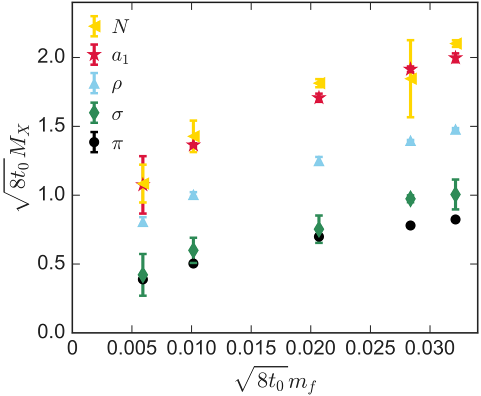
Near-conformal lattice studies

generically observe light scalar

Review: Witzel, arXiv:1901.08216

Requires reformulating

low-energy chiral effective field theory



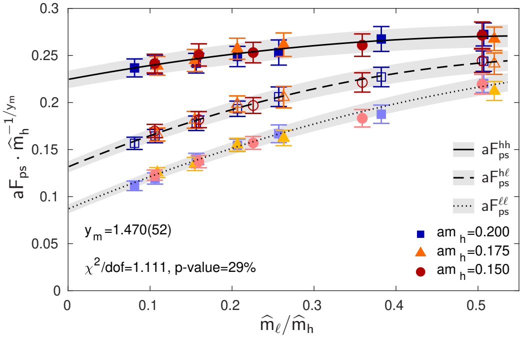
Scale separation from mass splitting

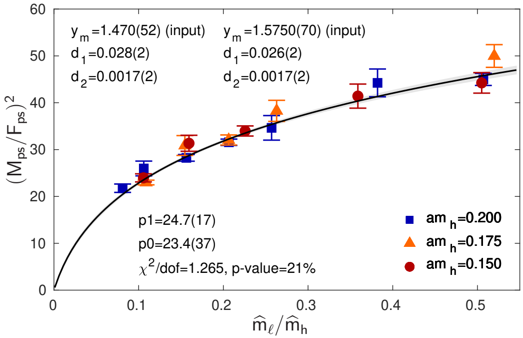
arXiv:2007.01810

Build in tunable scale separation

by considering four lighter flavours and six heavier flavours

Results exhibit conformal hyperscaling (**left**) and can be fit to dilaton-χPT (**right**)



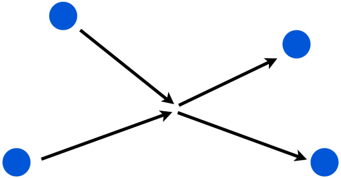


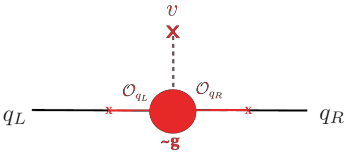
Promising directions for composite Higgs

Coming soon to DiRAC

Electroweak S parameter (vacuum polarization)

$W^{\pm}W^{\pm}$ scattering to test effective field theories





Another investigation being planned

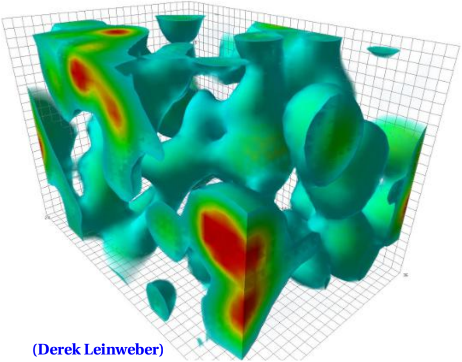
Baryon scaling dimensions for quark & lepton partial compositeness

Application: Supersymmetry and holography

Lattice field theory promises first-principles predictions
for strongly coupled supersymmetric QFTs

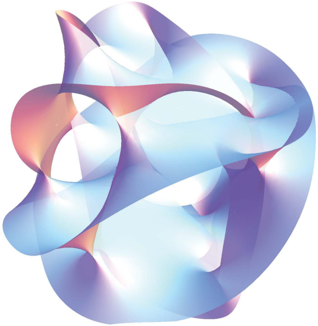
BSM

QFT



(Derek Leinweber)

Holography



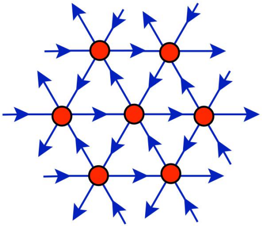
A brief history of lattice supersymmetry

Supersymmetries ‘square’ to infinitesimal translations, $\left\{Q^{\mathrm{I}}_{\alpha}, \overline{Q}^{\mathrm{J}}_{\dot{\alpha}}\right\} = 2\delta^{\mathrm{IJ}}\sigma^{\mu}_{\alpha\dot{\alpha}}P_{\mu}$

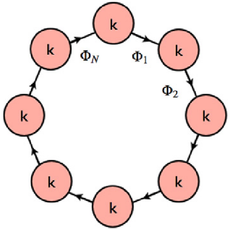
$\longrightarrow$ **do not exist** in discrete space-time

Solution: Reformulate theory to preserve subset of supersymmetries

$\Longrightarrow$ recover others in continuum limit



Review:
Catterall–Kaplan–Ünsal,
arXiv:0903.4881



Testing holographic duality

Holographic duality conjecture

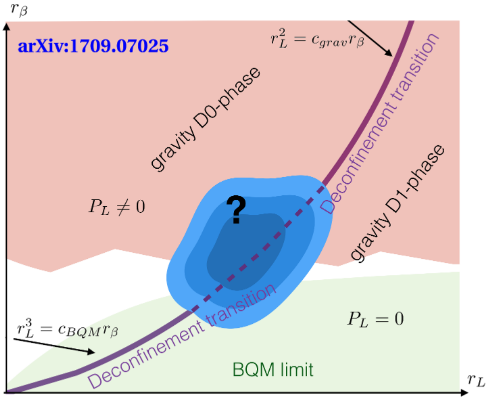
Thermodynamics of supersymmetric QFT $\longleftrightarrow$ black holes in dual supergravity

2d example: For decreasing r_L
at low $t = 1/r_\beta$ and large N

homogeneous black string (D1)
$\longrightarrow$ localized black hole (D0)

$\updownarrow$

"spatial deconfinement"
signalled by Wilson line



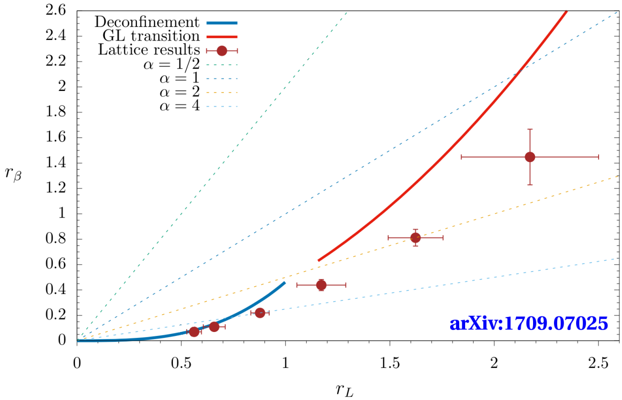
Two-dimensional super-Yang–Mills phase diagram

Lattice calculations with gauge groups up to SU(16)

Map out transitions in spatial Wilson line

Overall consistent with holography

At low temperatures (larger r_β)
harder to control uncertainties



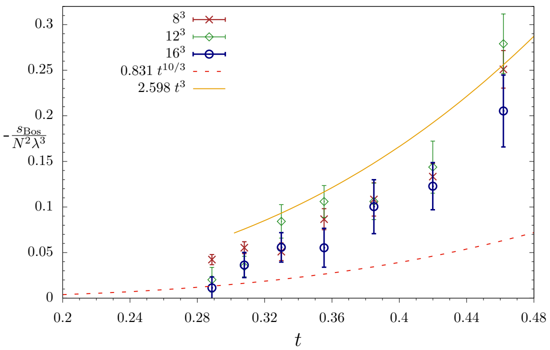
Three-dimensional super-Yang–Mills

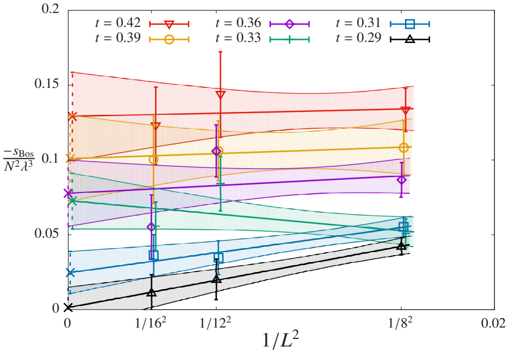
arXiv:2010.00026

Moving up to 3d thermodynamics with gauge group SU(8)

For low $t \lesssim 0.3$ dual black hole energy approaches holographic $\propto t^{10/3}$

First continuum extrapolations attempted, uncertainties still hard to control



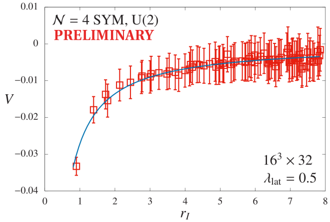


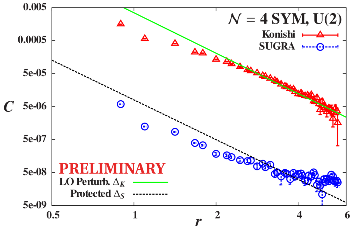
Promising directions for lattice supersymmetry

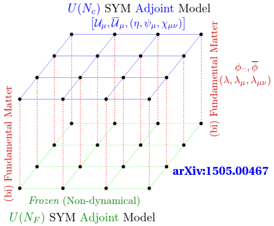
Super-Yang–Mills in four dimensions

The conformal field theory of the original AdS/CFT correspondence

Work underway on static potential, scaling dimensions and more [2102.06775]



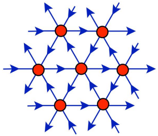




Super-QCD in lower dimensions

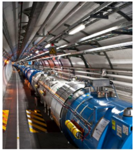
Quiver construction preserves subset of supersymmetries for $d < 4$

Outlook: Broad applications beckon

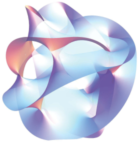


Lattice field theory is a broadly applicable tool
to study strongly coupled quantum field theories

Composite dark matter and gravitational waves
$\longrightarrow$ Colloquium by Kimmy Cushman, 22 April



Composite Higgs and near-conformal dynamics



Supersymmetry and holographic duality
$\longrightarrow$ Longer overview, 18 January
$\longrightarrow$ Colloquium by Raghav Govind Jha, 25 February

Thanks for your attention!

Any further questions?

Funding and computing resources

UK Research and Innovation

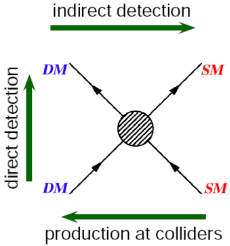
Liverpool Theoretical Physics

Department of Energy · United States of America

USQCD

Backup: Thermal freeze-out for relic density

Requires non-gravitational interactions with known particles



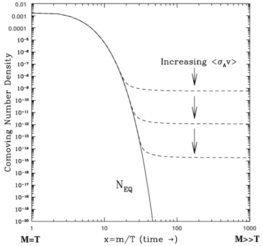
$\text{DM} \longleftrightarrow \text{SM}$ for $T \gtrsim M_{DM}$

$\text{DM} \longrightarrow \text{SM}$ for $T \lesssim M_{DM}$

$\Longrightarrow$ rapid depletion of Ω_{DM}

Hubble expansion

$\Longrightarrow$ dilution $\longrightarrow$ freeze-out



$2 \to 2$ scattering relates coupling and mass, $200\alpha \sim \dfrac{M_{DM}}{100\text{ GeV}}$

Strong $\alpha \sim 16 \longrightarrow$ 'natural' mass scale $M_{DM} \sim 300$ TeV

Smaller $M_{DM} \gtrsim 1$ TeV possible from $2 \to n$ scattering or asymmetry

Backup: Two roads to natural asymmetric dark matter

Idea: Dark matter relic density related to baryon asymmetry

$$\Omega_D \approx 5\Omega_B$$
$$\implies M_D n_D \approx 5 M_B n_B$$

$n_D \sim n_B \quad \implies \quad M_D \sim 5M_B \approx 5$ GeV

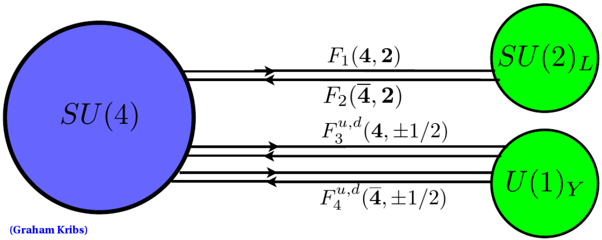
High-dim. interactions relate baryon# and DM# violation

$M_D \gg M_B \quad \implies \quad n_B \gg n_D \sim \exp\left[-M_D/T_s\right] \qquad T_s \sim 200$ GeV

EW sphaleron processes above T_s distribute asymmetries

Both require non-gravitational interactions with known particles

Backup: More details about SU(4) Stealth Dark Matter



(Graham Kribs)

Field	$SU(N_D)$	$(SU(2)_L, Y)$	Q
$F_1 = \begin{pmatrix} F_1^u \\ F_1^d \end{pmatrix}$	$\mathbf{N}$	$(\mathbf{2}, 0)$	$\begin{pmatrix} +1/2 \\ -1/2 \end{pmatrix}$
$F_2 = \begin{pmatrix} F_2^u \\ F_2^d \end{pmatrix}$	$\bar{\mathbf{N}}$	$(\mathbf{2}, 0)$	$\begin{pmatrix} +1/2 \\ -1/2 \end{pmatrix}$
F_3^u	$\mathbf{N}$	$(\mathbf{1}, +1/2)$	$+1/2$
F_3^d	$\mathbf{N}$	$(\mathbf{1}, -1/2)$	$-1/2$
F_4^u	$\bar{\mathbf{N}}$	$(\mathbf{1}, +1/2)$	$+1/2$
F_4^d	$\bar{\mathbf{N}}$	$(\mathbf{1}, -1/2)$	$-1/2$

Mass terms $\quad m_V\,(F_1F_2 + F_3F_4) + y\,(F_1\cdot HF_4 + F_2\cdot H^\dagger F_3) + \text{h.c.}$

Vector-like masses evade Higgs-exchange direct detection bounds

Higgs couplings $\longrightarrow$ charged meson decay before Big Bang nucleosynthesis

Both required $\longrightarrow$ four flavours

Backup: More details about form factors

Photon exchange via electromagnetic form factors

Interactions suppressed by powers of confinement scale $\Lambda \sim M_{DM}$

Dimension 5: Magnetic moment $\longrightarrow \; (\overline{X}\sigma_{\mu\nu}X)\; F^{\mu\nu}/\Lambda$

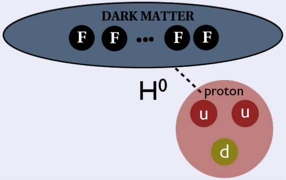
Dimension 6: Charge radius $\longrightarrow \; (\overline{X}X)\; v_\mu \partial_\nu F^{\mu\nu}/\Lambda^2$

Dimension 7: Polarizability $\longrightarrow \; (\overline{X}X)\; v_\mu v_\nu F^{\mu\alpha} F_\alpha^{\ \nu}/\Lambda^3$

Higgs exchange via scalar form factors

Higgs couples through σ terms $\langle B \,|\, m_\psi \overline{\psi}\psi \,|\, B\rangle$

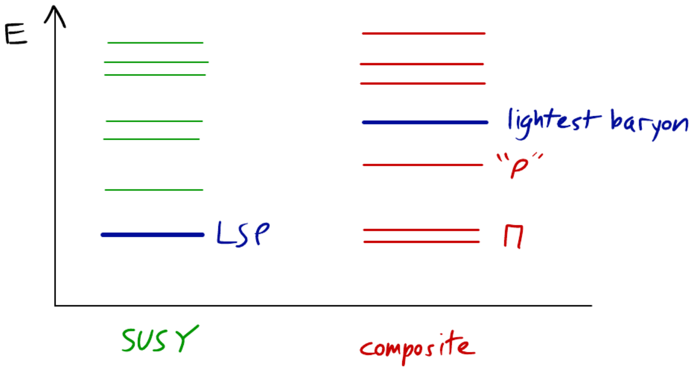
Produces rapid charged 'Π' decay
needed for Big Bang nucleosynthesis



Backup: Stealth Dark Matter at colliders

arXiv:1809.10184

The dark matter is the only stable composite particle, **not** the lightest



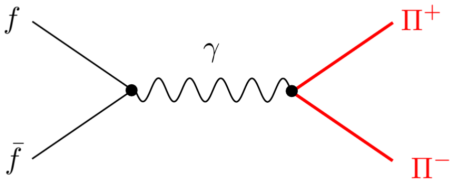
Main constraints from much lighter **charged** "Π"

$\longrightarrow$ standard 'missing energy' searches not efficient

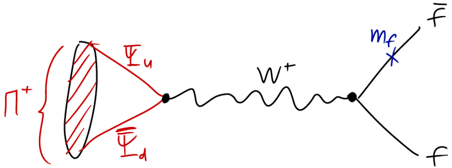
Backup: Stealth Dark Matter collider detection

arXiv:1809.10184

Production



Decay



"Particularly tricky" at the LHC: Recent bounds $M_\Pi \gtrsim 130$ GeV
similar to $M_\Pi \gtrsim 100$ GeV from LEP searches for SUSY tau-partner

Lattice calculation of $M_{DM}/M_\Pi \longrightarrow M_{DM} \gtrsim 300$ GeV

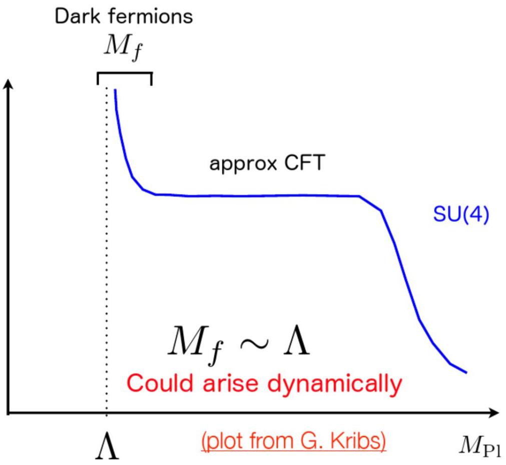
More form factors to compute: $F_1(4M_\Pi^2)$ for Π and decay constant F_Π

Backup: Stealth Dark Matter mass scales

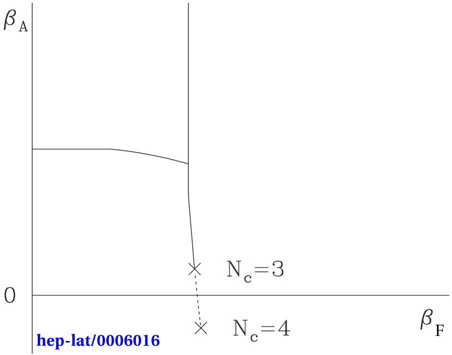
Lattice studies focus on $m_\psi \simeq \Lambda_{DM}$ where effective theories least reliable

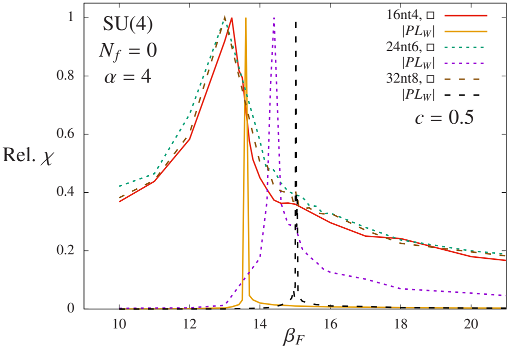
$m_\psi \simeq \Lambda_{DM}$ could arise dynamically

Collider constraints on M_{DM}
become stronger as m_ψ decreases



Backup: Pure gauge checks — Bulk and thermal transitions

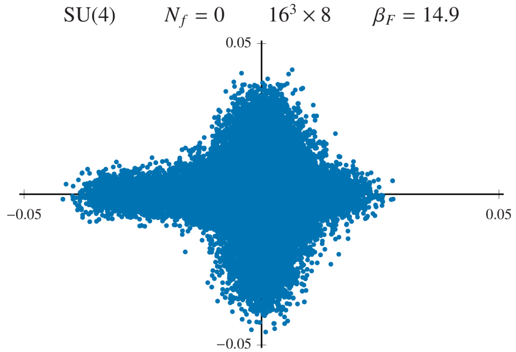


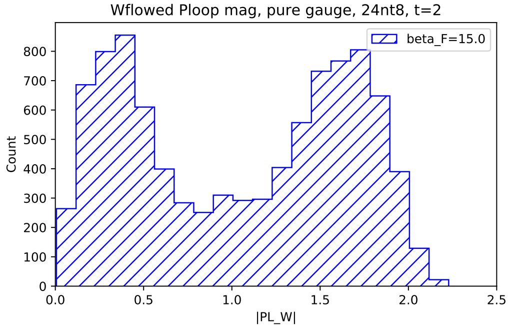


Try to avoid bulk transition for small N_T $\longrightarrow$ use $\beta_A = -\beta_F/4$

Still need $N_T > 4$ for clear separation between bulk & thermal transitions

Backup: Pure gauge checks — Order of thermal transition



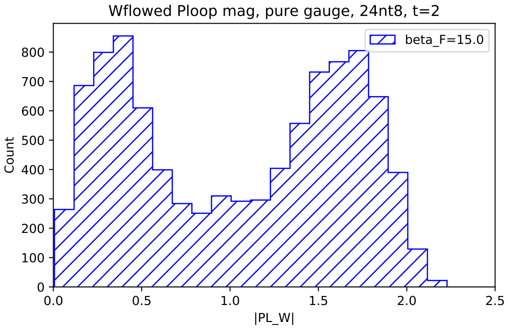


Two peaks in Polyakov loop magnitude histogram $\longrightarrow$ first-order transition ✓

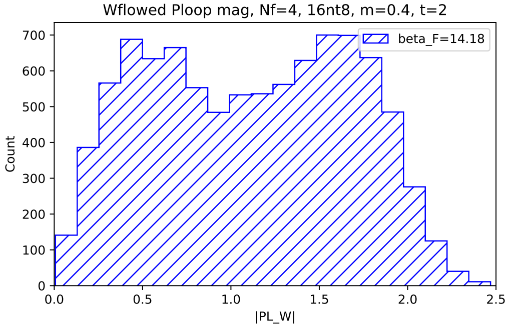
Hysteresis not clearly visible even in pure-gauge case

Backup: Order of thermal transition with dynamical fermions

Pure gauge



Four flavours



Two peaks in Polyakov loop magnitude histogram $\longrightarrow$ first-order transition ✓

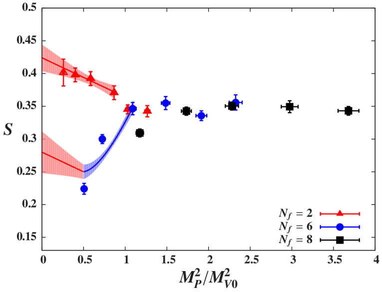
Hysteresis not clearly visible even in pure-gauge case

Backup: S parameter on the lattice

$$\mathcal{L}_\chi \supset \frac{\alpha_1}{2} g_1 g_2 B_{\mu\nu} \mathrm{Tr}\left[U \tau_3 U^\dagger W^{\mu\nu}\right] \longrightarrow$$

Lattice vacuum polarization calculation provides $S = -16\pi^2\alpha_1$

Non-zero masses and chiral extrapolation needed due to finite lattice volume



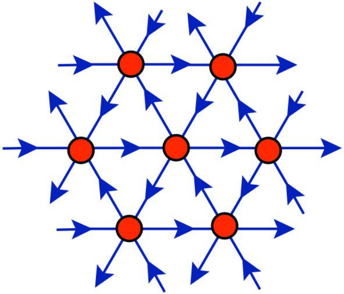
$S = 0.42(2)$ for $N_F = 2$ matches scaled-up QCD

Larger $N_F \longrightarrow$ significant reduction

Extrapolation to correct zero-mass limit
becomes more challenging

Backup: Five links in four dimensions $\longrightarrow$ A_4^* lattice

A_4^* $\sim$ 4d analog of 2d triangular lattice



Basis vectors linearly dependent and non-orthogonal

Large S_5 point group symmetry

S_5 irreps precisely match onto irreps of twisted $\text{SO}(4)_{\text{tw}}$

$$\psi_a \longrightarrow \psi_\mu, \quad \overline{\eta} \qquad \text{is} \qquad \mathbf{5} \longrightarrow \mathbf{4} \oplus \mathbf{1}$$

$$\chi_{ab} \longrightarrow \chi_{\mu\nu}, \quad \overline{\psi}_\mu \qquad \text{is} \qquad \mathbf{10} \longrightarrow \mathbf{6} \oplus \mathbf{4}$$

$S_5 \longrightarrow \text{SO}(4)_{\text{tw}}$ in continuum limit restores $\mathcal{Q}_a$ and $\mathcal{Q}_{ab}$

Backup: Public code for lattice $\mathcal{N} = 4$ SYM

so that the full improved action becomes

$$S_{\text{imp}} = S'_{\text{exact}} + S_{\text{closed}} + S'_{\text{soft}} \tag{18}$$

$$S'_{\text{exact}} = \frac{N}{4\lambda_{\text{lat}}} \sum_n \text{Tr}\Bigg[-\overline{\mathcal{F}}_{ab}(n)\mathcal{F}_{ab}(n) - \chi_{ab}(n)\mathcal{D}^{(+)}_{[a}\psi_{b]}(n) - \eta(n)\overline{\mathcal{D}}^{(-)}_a\psi_a(n) + \frac{1}{2}\left(\overline{\mathcal{D}}^{(-)}_a\mathcal{U}_a(n) + G\sum_{a\neq b}\left(\det\mathcal{P}_{ab}(n) - 1\right)\mathbb{I}_N\right)^2\Bigg] - S_{\text{det}}$$

$$S_{\text{det}} = \frac{N}{4\lambda_{\text{lat}}} G \sum_n \text{Tr}\left[\eta(n)\right] \sum_{a\neq b}\left[\det\mathcal{P}_{ab}(n)\right] \text{Tr}\left[\mathcal{U}_b^{-1}(n)\psi_b(n) + \mathcal{U}_a^{-1}(n+\widehat{\mu}_b)\psi_a(n+\widehat{\mu}_b)\right]$$

$$S_{\text{closed}} = -\frac{N}{16\lambda_{\text{lat}}} \sum_n \text{Tr}\left[\epsilon_{abcde}\ \chi_{de}(n+\widehat{\mu}_a+\widehat{\mu}_b+\widehat{\mu}_c)\overline{\mathcal{D}}^{(-)}_c\chi_{ab}(n)\right],$$

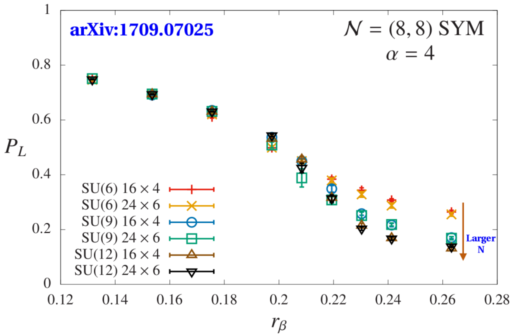
$$S'_{\text{soft}} = \frac{N}{4\lambda_{\text{lat}}} \mu^2 \sum_n \sum_a \left(\frac{1}{N}\text{Tr}\left[\mathcal{U}_a(n)\overline{\mathcal{U}}_a(n)\right] - 1\right)^2$$

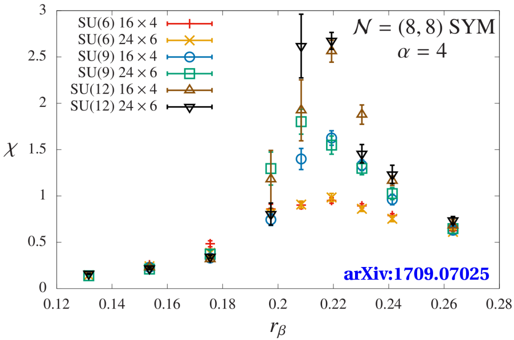
$\gtrsim$100 inter-node data transfers in the fermion operator — non-trivial. . .

Public parallel code to reduce barriers to entry: github.com/daschaich/susy

Evolved from MILC QCD code, user guide in arXiv:1410.6971

Backup: Spatial deconfinement transition signals



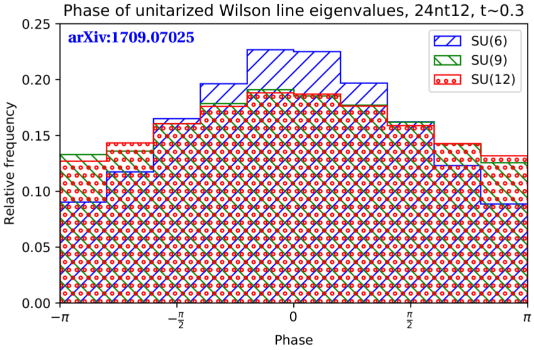


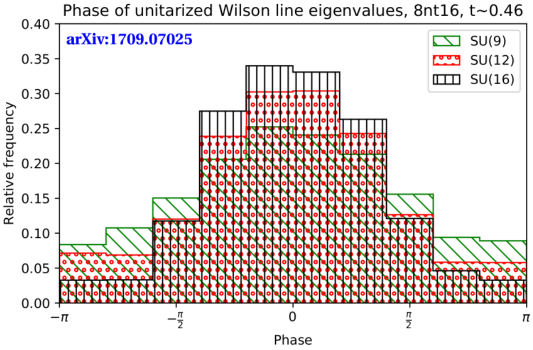
Peaks in Wilson line susceptibility match change in its magnitude |PL|,
grow with size of SU(N) gauge group, comparing $N = 6, 9, 12$

Agreement for 16×4 vs. 24×6 lattices (aspect ratio $\alpha = r_L / r_\beta = 4$)

Backup: 2d $\mathcal{N} = (8, 8)$ SYM Wilson line eigenvalues

Check 'spatial deconfinement' through Wilson line eigenvalue phases



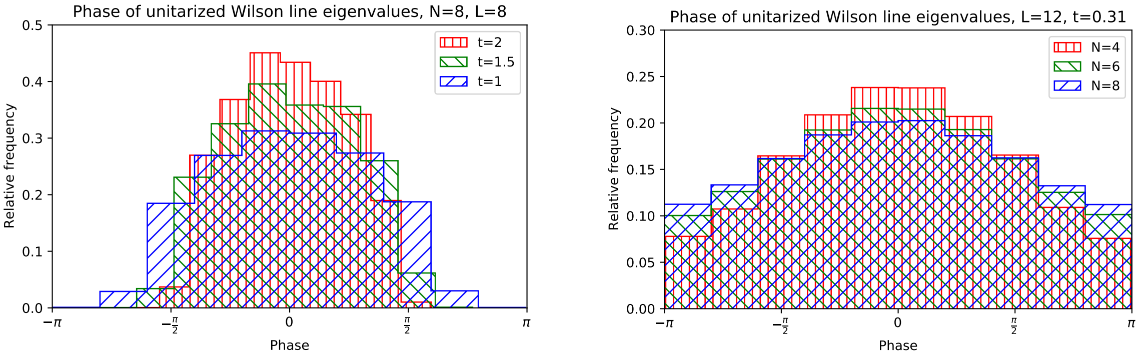


Left: $\alpha = 2$ distributions more extended as N increases $\longrightarrow$ D1 black string

Right: $\alpha = 1/2$ distributions more compact as N increases $\longrightarrow$ D0 black hole

Backup: 3d $\mathcal{N} = 8$ SYM Wilson line eigenvalues

Check 'spatial deconfinement' through Wilson line eigenvalue phases



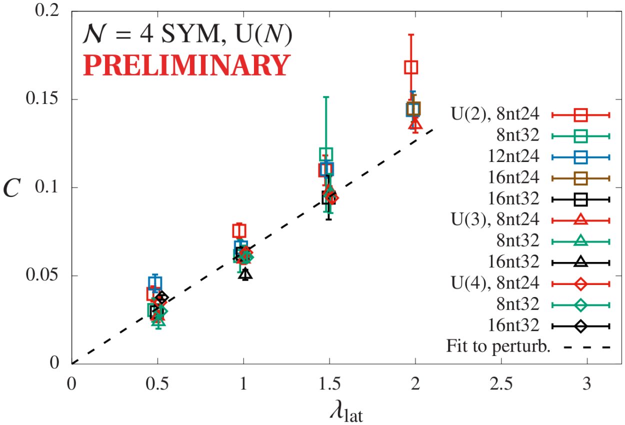
Left: High-temperature U(8) 8^3 distributions more compact as t increases

Right: Low-temperature U(N) 12^3 distributions more uniform as N increases

Backup: Coupling dependence of Coulomb coefficient

Continuum perturbation theory $\longrightarrow$ $C(\lambda) = \lambda/(4\pi) + \mathcal{O}(\lambda^2)$

Holography $\longrightarrow$ $C(\lambda) \propto \sqrt{\lambda}$ for $N \to \infty$ and $\lambda \to \infty$ with $\lambda \ll N$



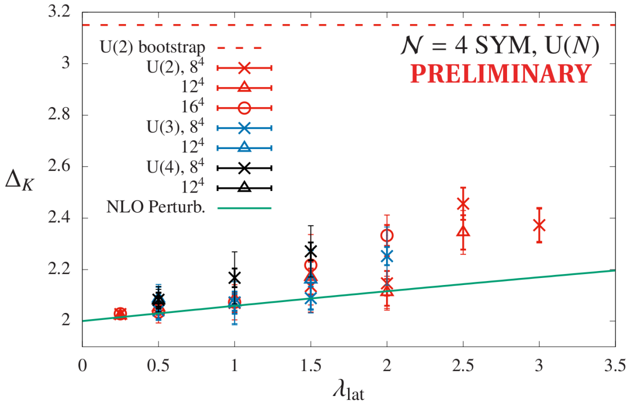
For $\lambda_{\text{lat}} \leq 2$, consistent with leading-order perturbation theory

Backup: Preliminary Δ_K results from Monte Carlo RG

Analyzing both $\mathcal{O}_K^{\text{lat}}$ and $\mathcal{O}_S^{\text{lat}}$

Imposing protected $\Delta_S = 2$
$\longrightarrow \Delta_K(\lambda)$ looks perturbative

Systematic uncertainties from
different amounts of smearing



Complication from twisting $SO(4)_R \subset SO(6)_R$

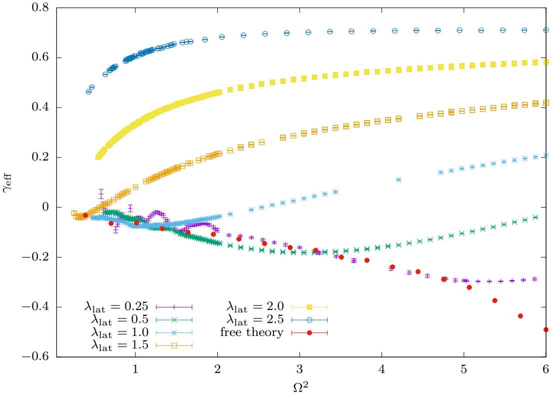
$\mathcal{O}_K^{\text{lat}}$ mixes with $SO(4)_R$-singlet part of $SO(6)_R$-nonsinglet $\mathcal{O}_S$
$\longrightarrow$ disentangle via variational analyses

Backup: Fermion bilinear anomalous dimension

arXiv:2102.06775

Fermion op. eigenvalues predict 'mass' anomalous dimension of fermion bilinear

Should vanish $\longrightarrow$ test discretization and finite-volume effects in lattice calcs



Scale-dependent 'effective anom. dim.' due to broken conformality

Recover true critical exponent at low energy scale $\Omega^2 \ll 1$

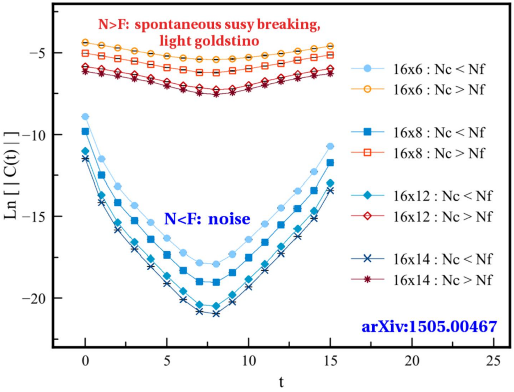
$0.25 \leq \lambda_{\text{lat}} \leq 2.5$, with even free theory sensitive to lattice effects

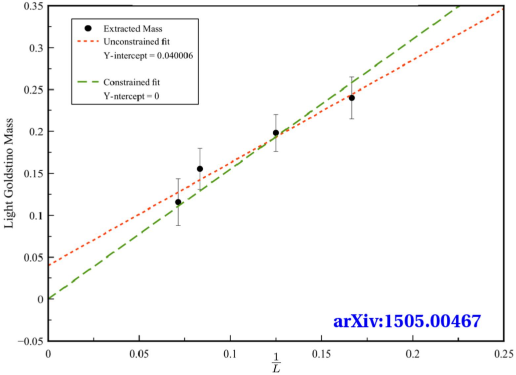
Backup: Dynamical susy breaking in 2d lattice superQCD

U(N) superQCD with F fundamental hypermultiplets

Observe spontaneous susy breaking only for $N > F$, as expected

Catterall–Veernala, arXiv:1505.00467





Backup: More on dynamical susy breaking

Spontaneous susy breaking means $\langle 0\,|H|\,0\rangle > 0$ or equivalently $\langle \mathcal{Q}\mathcal{O}\rangle \neq 0$

Twisted superQCD auxiliary field e.o.m. $\longleftrightarrow$ Fayet–Iliopoulos D-term potential

$$d = \overline{\mathcal{D}}_a \mathcal{U}_a + \sum_{i=1}^{F} \phi_i \overline{\phi}_i - r\mathbb{I}_N \qquad \longleftrightarrow \qquad \text{Tr}\left[\left(\sum\nolimits_i \phi_i \overline{\phi}_i - r\mathbb{I}_N\right)^2\right] \in H$$

Have $F \times N$ scalar vevs to zero out $N \times N$ matrix

$\longrightarrow N > F$ suggests susy breaking, $\langle 0\,|H|\,0\rangle > 0 \longleftrightarrow \langle \mathcal{Q}\eta\rangle = \langle d\rangle \neq 0$