































Finite Field

- \square Also called *Galois Field*, denoted by GF(N)
- \square $N = p^k$, p is a prime number, and k an integer
- □ Each element is a k-tuple
- □ There are exactly *N* elements in the field (0, 1, ..., N-1) or $(0, 1, \alpha, \alpha^2, \alpha^3, ..., \alpha^{N-1})$ where , α primitive element

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Exam	ple G	F(4)		
Let o	$\alpha^2 = \beta$			
Thus	s we hav	ve 4 elements	$0, 1, \alpha$ and β	
	GF(4)	2-tuple of GF(2)	Polynomial Representation	
	0	0.0	0	
	1	0 1	1	
	α	10	X	
	ß	11	(x+1)	



Addition

The addition of any two elements in GF(4), can be performed as addition of polynomials over GF(2)

For example, consider $\alpha + \beta$

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\alpha = x and \beta = x + 1
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 $\alpha + \beta = x + x + 1 = 2 x + 1$

Since 2 x = 0 over $GF(2^2)$

 $\alpha + \beta = x + x + 1 = 1$

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2-Bit Multiplier Galois Expression

- Inputs partitioned as $X = (x_1, x_0), Y = (y_1, y_0)$
- Outputs partitioned as $Z_1 = (z_3, z_2), Z_2 = (z_1, z_0)$
- GFMODD yields

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 $Z_{1} = \alpha X^{3}Y^{2} + \alpha XY^{2} + X^{3}Y + \alpha X^{2}Y + \beta XY + \beta X^{3}Y^{3} + \alpha X^{2}Y^{3} + XY^{3}$ $Z_{2} = \beta X^{2}Y^{2} + \beta X^{2}Y + \beta XY^{2} + \alpha XY$

Gfxpress (TCAD 2008) uses an algorithm using the Polynomial description as input to synthesize low power designs. This has been recently augmented to incorporate soft Error correction

Powe	Power Aware Soft Error Robust							
Multi	plier Desig	n		10 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1				
	Synopsys Tool Only		Our Tool and Synopsys tool					
Pimitive Poly	(Area,Delay,Power)	(EXOR,AND)	(Area,Delay,Power)					
131	(30866.1,7.7,39.1)	(48,49)	(1699.9,1.3,2.4)					
137	(30356.5,8.3,39.9)	(50,49)	(1709.6, 1.6, 2.5)					
143	(33375.6,8.0,40.3)	(67,49)	(1925.7,1.9,3.0)					
145	(33469.1,8.1,42.3)	(50,49)	(1703.1,1.7,2.5)	Area in 10 ⁻⁶ mm ² ,				
157	(35020.6,7.4,42.7)	(65,49)	(1964.4,2.1,3.1)	delay in ns				
167	(34978.7,7.7,39.7)	(65,49)	(1964.4,1.7,3.2)					
171	(39023.2,8.5,49.1)	(64,49)	(1906.3,1.4,2.9)	power in microw at 1.8				
185	(34797.9,8.7,42.1)	(66,49)	(2003.1,1.4,3.0)					
191	(34843.1,8.8,41.0)	(75,49)	(2077.3,1.8,3.2)					
193	(30321.0,8.0,38.8)	(48,49)	(1725.7,3.2,2.7)					
203	(33875.6,8.0,42.0)	(64,49)	(1922.5,1.5,3.0)					
211	(33440.1,8.2,40.3)	(64,49)	(1935.4,1.7,3.1)					
213	(33488.5,8.0,38.9)	(65,49)	(2009.6,1.8,3.3)					
229	(34578.6,8.2,41.3)	(67,49)	(1980.5,1.8,3.2)					
239	(34123.9,9.9,41.5)	(72,49)	(2041.8,1.6,3.1)					
241	(32298.3,8.7,36.7)	(61,49)	(1899.9,1.6,3.0)					
247	(36704.1,7.2,45.0)	(72,49)	(2106.3, 1.6, 3.2)					
253	(33782.0,8.3,41.3)	(74,49)	(2106.3,2.0,3.2)	and the second second				
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Auu	er Desigi	.1	1000	The second second second
	Synopsys Only		With our technique	
Adder	(area,delay,power)	(a,m)	(area,delay,power)	
2-bit	(119.3,0.3,115.7)	(4,5)	(106.4,0.4,113.1)	
3-bit	(197.8,0.7,214.3)	(11,6)	(216.1,0.81,241)	
4-bit	(274.2,1.21,301.8)	(17,8)	(316.1,1.08,378.2)	
5-bit	(383.8,1.22,468.5)	(23,10)	(416.1,1.4,515.8)	
6-bit	(493.5,1.58,590)	(29,12)	(509.6, 1.72, 627.5)	
7-bit	(638.7,1.61,791.1)	(35,14)	(609:3,2.04,752.8)	
8-bit 🔨	 (687:0,2.0,809:4)- -	· -(41,16) -	(709.6,2.38,879)	
9-bit	(11877,3.7,13800)	(47,18)	(809.6,2.69,1000)	
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