RAILWAY NOISE AND VIBRATION: THE USE OF APPROPRIATE MODELS TO SOLVE PRACTICAL PROBLEMS

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Railways are an environmentally friendly means of transport that offer many advantages for sustainable development. However, noise and vibration remain a major reason for objections to new lines or network expansions. Railways will never be silent but, it is nevertheless important to reduce their noise and vibration as much as possible while not adding unnecessary cost or complication to their construction and operation. To achieve this it is essential to understand the sources of noise and vibration and the parameters that can influence them in order to propose cost-effective mitigation measures. Theoretical models are an important part of this process. Such models should be of sufficient detail to cover the most relevant parameters in a reliable way, but not over-complicated in order not to lose insight. This approach of developing appropriate models for the purpose of designing mitigation measures is illustrated through several examples related to rolling noise, aerodynamic noise and ground vibration.

1. Introduction

Railways are undergoing a renaissance in many countries as they become seen as a more sustainable and environmentally-friendly means of transport than road or air. As new lines are planned and opened in many countries, and as line speeds and traffic densities are increased, the resulting noise and vibration take on a growing importance. Local residents use noise and vibration as a means of objecting to railway developments and consequently expensive mitigation measures have to be included in new designs.

Railways will never be silent; to move large quantities of passengers or freight at high speeds will inevitably involve the generation of significant noise and vibration through both mechanical vibration and aerodynamic noise. Nevertheless, it is clearly important to reduce its impact as much as possible while not adding unnecessary cost or complication to the construction and operation of the railways. To achieve this, it is essential to understand the sources of noise and vibration and the parameters that can influence them, in order to propose cost-effective mitigation measures.

Theoretical models have been developed for many railway noise phenomena.[1] Ultimately, the purpose in developing such a model is in order to solve a problem, so a good test of whether the model is appropriate is whether it can be used effectively to address the corresponding problem. If a model is too simple it may not reliably represent the physics of the situation. Consequently it may give an incorrect diagnosis, or may give misleading recommendations or expectations. On the other hand, if a model is too complex it will be difficult to quantify the various input parameters in suffi-

cient detail and validation will be problematic. Moreover, it will be difficult to perform parametric studies with such complex models and insight will not be gained into the underlying phenomena. The development of advanced computing facilities makes more complex modelling possible, but it is not always desirable to consider ever more-complex models.

Models should thus be of sufficient detail to cover the most relevant parameters in a reliable way, but not over-complicated in order not to lose insight. This approach of developing appropriate models for the purpose of designing mitigation measures is illustrated here through several examples from the author's experience; these cover aspects of rolling noise, aerodynamic noise and ground vibration.

2. Rolling noise

The dominant source of noise from operational railways in most situations is that due to the rolling of the wheel along the rail. The surface roughness (unevenness) of the wheel and rail induces relative vibration of the wheel and rail, the vibration amplitude of each component depending on their dynamic properties. The resultant vibration then radiates noise.

Modelling of rolling noise began in the 1970s with Remington's work.[2,3] In this, analytical models were used for the wheel and rail impedances and for radiation efficiencies. His work took account of many features that are still considered as important: relative displacement excitation by the roughness, the contact filter effect, track decay rates, etc. Subsequently this basic model was developed and extended to include other features that were found to be significant[4-9], and this was implemented in the TWINS (Track-Wheel Interaction Noise Software) package on behalf of the European railways and validated using extensive field tests.[10-12]

2.1 The TWINS model

Although the TWINS rolling noise model was developed over 20 years ago, it is still widely used in the railway industry and considered as state-of-the-art. It comprises a series of sub-models, each chosen to provide an appropriate level of detail. Figure 1 shows the model schematically.
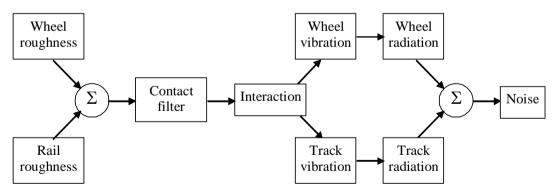

Figure 1. Model for rolling noise generation.

2.1.1 Roughness excitation

For roughness of wavelength λ (in m), the frequency of excitation, in Hz, is given by $f = V/\lambda$, where V is the train speed (in m/s). Relevant wavelengths are between about 5 and 200 mm and typical amplitudes are of the order of microns. Wavelengths that are short compared with the contact patch length (typically 10-15 mm) are attenuated in their excitation of the system, giving rise to the so-called contact filter effect. Remington originally derived an analytical model of the contact filter[2] and later developed a numerical model based on a set of discrete point-reacting springs within the contact zone.[13] When used with measured roughness data, the latter model gives a smaller filter effect at short wavelengths.[14] In practical application tabulated values can also be used.[1]

2.1.2 Wheel vibration

The wheel is a lightly damped resonant structure. It has a complex geometry which makes it less amenable to analytical modelling than the rail and so finite element (FE) models are more appropriate. Nevertheless, in most cases the axisymmetric geometry can be taken into account, allowing the use of axi-harmonic FE models. These use the 2D cross-sectional geometry and allow the modal order (number of nodal diameters, n) to be included explicitly. Examples of modeshapes are shown in Figure 2. The one-nodal-circle axial modes and the predominantly radial modes are most important for rolling noise, being excited by a radial force.

If a full three-dimensional FE model is used, the modal order has to be identified by careful inspection of the modeshape. Moreover, all modes of vibration with $n \geq 1$ appear in pairs with identical natural frequency and their angular orientation is arbitrary. Where a wheel has a non-axisymmetric geometry, such as where brake discs are fitted to the web, a full three-dimensional FE model is required. However, even here, it is often sufficient to represent the wheel by an equivalent axi-symmetric model.
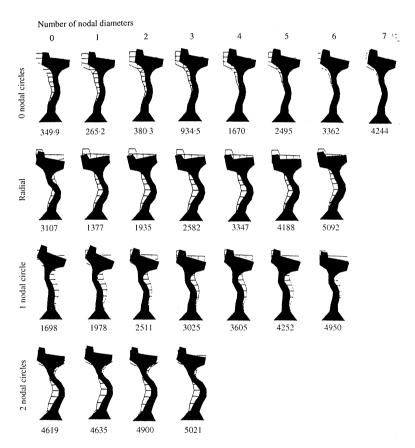

Number of nodal diameters

	0	1	2	3	4	5	6	7
0 nodal circles	349·9	265·2	380·3	934·5	1670	2495	3362	4244
Radial	3107	1377	1935	2582	3347	4188	5092	
1 nodal circle	1698	1978	2511	3025	3605	4252	4950	
2 nodal circles	4619	4635	4900	5021				

Figure 2. Examples of modeshapes of a railway wheel, showing cross-section through the wheel. Figures are calculated natural frequencies in Hz.[1]

2.1.3 Rail vibration

Unlike the wheel, the rail is effectively an infinite waveguide which sustains propagating waves rather than resonances. The decay rate of these waves with distance along the rail determines the length of rail that vibrates with each wheel and thereby controls the radiated power. The vertical dynamics of the rail can be adequately modelled using an analytical model of a beam on a two-layer elastic support; rail pads and ballast are represented by elastic layers while the sleepers are represented by their mass or by a transverse beam. The lateral dynamic behaviour is less well represented by this model as torsion makes it more flexible than the simple beam implies. At higher frequency, above about 1 kHz for the lateral direction, the cross-sectional deformation of the rail section also

becomes important. To account for these effects a full FE model could be used in principle but would be very computer intensive, as a very long section of track would be required to avoid end effects. FE-based approaches making use of periodic structure theory[7] or waveguide finite elements[15] can be used but it is also effective to use multiple beam models[16,17,18] which are much simpler than FE models.

2.1.4 Dynamic interaction

The surface roughness introduces a relative displacement between the wheel and rail in the vertical direction. The actual motion of the wheel and rail at the contact depends on whether the mobility of the rail (Y_R) or the wheel (Y_W) has the largest amplitude. The Hertzian contact spring (also described by its mobility Y_C) also plays a role. The equations of motion for interaction via a single degree of freedom at circular frequency ω are:

$$v_R = \frac{i\omega r Y_R}{Y_R + Y_W + Y_C} \; ; \; v_W = \frac{-i\omega r Y_W}{Y_R + Y_W + Y_C} \tag{1}$$

where v_R and v_W are the vertical velocities of the rail and wheel at the contact point and r is the roughness amplitude. It is found that the rail has the largest mobility over most of the region 100-1000 Hz; consequently, $v_R \approx i\omega r$ in this region. The wheel response is greater at high frequencies, where a number of lightly damped resonances occur with strong radial components, see Figure 2.

2.1.5 Sound radiation

To calculate the sound radiation from a vibrating object, numerical techniques such as boundary elements (BE) and finite/infinite elements are increasingly used. Nevertheless, they are generally limited to the calculation of sound radiation at low frequencies or for simple geometries.

It is useful to separate the acoustic radiation problem into estimation of the radiated power and a separate estimation of directivity. The power W in a frequency band can be found from the surface-averaged mean-square velocity $\left\langle \overline{v^2} \right\rangle$ using

$$W = \rho c S \sigma \left\langle \overline{v^2} \right\rangle \tag{2}$$

where ρc is the characteristic acoustic impedance, S is the surface area and σ is the radiation efficiency. It has been found that the radiation efficiency of a wheel can be adequately represented by simple semi-analytical formulae that are obtained by curve-fitting to the results from axi-harmonic BE calculations.[1,19]

To obtain the sound pressure requires additional information about the directivity. However, the complex geometry of the train, bogie and ground and the air flow around the train mean that, even if the directivity can be correctly predicted in laboratory conditions, the actual situation will be different. Since the objective is usually to obtain the average sound pressure level during a pass-by, details of the directivity are less important and simple monopole or dipole directivities can be used to good effect.[19]

For the rail, again a full three-dimensional BE model of a length of rail would be too computationally intensive. However, as the wavelength of waves in the rail is generally much greater than the acoustic wavelength, a two-dimensional BE approach is found to be valid for all except very low frequencies.[20] Until recently, the effect of the proximity of the ground on the radiation from the rail has been neglected but the effects can be significant, as shown in a separate paper.[21]

An example of the contributions of wheel, rail and sleepers to the overall noise, as calculated by the TWINS model, is shown in Figure 3. These results are for an EMU running at 120 km/h on a track with resilient rail pads and in this case the rail is the most important source, the wheel contribution being about 7 dB lower overall. As the speed or roughness spectrum change, the relative

importance of different frequency bands changes, although the relative levels of each component within a given frequency band remain the same. Thus for higher speeds the importance of higher frequencies increases and the wheel component becomes more important in the overall level.
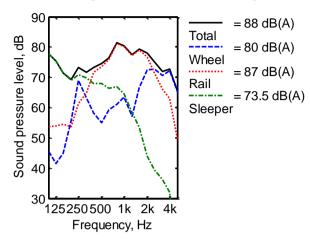

Figure 3. Example of components of rolling noise for an EMU at 120 km/h on track with soft pads. [22]

2.1.6 Slab track

Slab track is known to be noisier than ballasted track. One difference that is often cited is that the absorption of the ballast is missing in the case of a slab track. Measurements of the absorption of railway ballast show an absorption coefficient of up to about 0.6 at high frequencies, reducing at low frequencies, as shown in Figure 4. [23] This reduces the reflected sound from the wheel and rail to some extent but the overall difference in wayside sound pressure level for a track in free field is found to be approximately 1 dB.
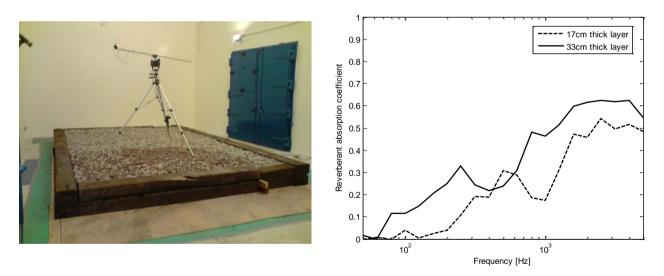

Figure 4. Measurement of diffuse incidence absorption coefficients of ballast in a reverberation room. Results are shown for two different thicknesses. [23]

A more important difference between slab track and ballasted track, however, is the track decay rate. The use of softer rail fasteners on slab track means that track decay rates are typically lower, resulting in higher noise levels (see Section 2.2 below). The actual effect depends on the type of rail fastener used, but can be several dB.

2.1.7 Example: effect of a track renewal on noise

As an example of the use of the model in interpreting measured results, results are presented of noise measured at a site in the UK. [22] One track had recently been renewed while the other re-

mained in its un-renewed state. Both had welded rail on concrete sleepers. An important feature is that the renewed track was fitted with 10 mm resilient rail pads normally fitted to modern track while the un-renewed track had thinner stiffer pads. The rail roughness and track decay rates were measured at the site. In addition, wheel roughness measurements were made on similar rolling stock. Predictions of the noise were found to be close to the measured values, as shown in Figure 5.

On the renewed track, with soft pads, the noise levels were 3-4 dB(A) greater than on the un-renewed track. This was found to be due to a reduction in track decay rates[24] which more than offset a small reduction in rail roughness. The wheel component of noise was found to be about 4 dB less than that of the track for the un-renewed track and 7 dB less than the track component on the renewed track, as already shown in Figure 3. These results suggest that for modern UK track and rolling stock, the track component of noise is of much greater importance than the wheel.
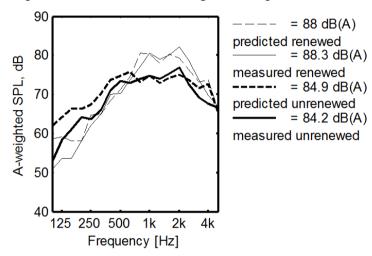

Figure 5. Noise from EMUs on a renewed track (thin lines) and an un-renewed track (thick lines), measured (——) and predicted ($- - -$) for 7.5 m from the track and a train speed of 110-120 km/h.[22]

2.2 Mitigation measures

2.2.1 Summary of possible measures

Following successful its validation[12] the rolling noise model can used to identify the parameters that can be modified in order to reduce noise. Thus it can be seen from Figure 1 that the noise can be controlled by reducing the roughness of the wheel and/or rail, depending on which is initially dominant. This reduces the vibration response of both wheel and track and the corresponding sound radiation in direct proportion to the reduction in combined roughness. Other changes lead to a reduction of the wheel or track noise so that it is important to identify the dominant component and to focus noise mitigation on this. However, if wheel and track are both important, mitigation measures are required for both components.

The parameter with the strongest influence on the amount of noise radiated by the rail is the rate of decay of vibration along the rail.[25] A change in decay rate from DR_u to DR_d (in dB/m) leads to a change in noise level (in dB), in the corresponding frequency band, of

$$\Delta L = 10\log_{10}(DR_u/DR_d) \tag{3}$$

A doubling of decay rate in a particular frequency band thus leads to a reduction of noise from the rail in that band by 3 dB. Measures that aim to introduce damping to the rail are therefore likely to be among the most effective in reducing track noise.

For the sound radiated by the wheel, the main contribution occurs at high frequency, usually between about 1.5 kHz and 5 kHz, as seen in Figure 4. The upper limit is determined by the roll-off of the contact filter. In this frequency region lightly damped wheel resonances are excited with a

large radial component. Due to asymmetry of the wheel cross-section, as seen in Figure 2, these modes also have large axial components on the wheel web, which radiate sound efficiently. Both radial and 1-nodal-circle axial modes are strongly excited. Their motions are coupled, so that both sets of modes contain both radial and web axial motion[6].

Finally, shielding measures can be applied to reduce the radiated sound. However, conventional noise barriers are expensive and visually intrusive. A fuller discussion of mitigation measures for rolling noise was given in [26,27].

2.2.2 Wheel and rail dampers

Wheel and rail damping treatments are now becoming more common. However, the prospect of having to test new prototypes in the field is very expensive. Therefore a recent German-French collaborative research project, STARDAMP, led by the German and French Railways, had the aim of developing laboratory test methods for the assessment of wheel and rail damping treatments. Combining these test methods with a prediction model allows the effect of damping treatments to be assessed in a transferable way.[28,29] The prediction model embodied in the Stardamp software has a similar functionality to the established TWINS model, but is designed for use by non-experts. It therefore has a reduced set of input parameters and many preset options.
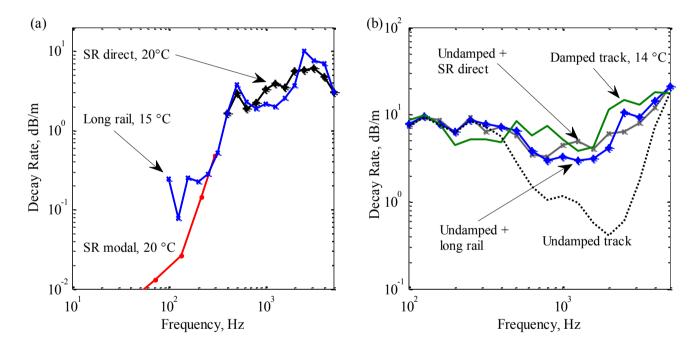

Figure 6. (a) Vertical decay rates measured on free rails fitted with rail dampers, DR_f. SR = short rail. (b) Vertical decay rates of undamped track, DR_u, track fitted with dampers, DR_d, and estimates of damped track obtained from equation (4).[29]

To assess the behaviour of rail dampers, the most important parameter is the track decay rate, as discussed above. In the STARDAMP method, the decay rate of the damped track, DR_d, is estimated by first measuring the decay rate of a free rail with dampers attached, DR_f. Results obtained for dampers fitted to a 6 m rail length and a 32 m length are compared in Figure 6(a). These results are then added to the decay rate of an undamped track to estimate the damped track decay rate:

$$DR_d \approx DR_u + DR_f \tag{4}$$

The result of this is shown in Figure 6(b) and compared with a direct measurement of DR_d from which it can be seen that quite good agreement is obtained.

2.3 Virtual testing

Noise limits for individual road vehicles and aircraft have been in place in Europe and elsewhere since the 1970s. For new trains, however, noise limits were not introduced in Europe until 2002. These are defined in Technical Specifications for Interoperability (TSI).[30] Tests are included for trains running at constant speed, stationary, and accelerating. As these tests are expensive to carry out, there is interest in developing virtual testing, under which the tests are, at least partially, replaced by calculations. The EU project ACOUTRAIN[31] has been investigating the potential for such tests.

In this context, a different type of prediction model is required. This should combine source data from a variety of components, together with the geometry of the train to predict the sound pressure level at various positions around a stationary train and at fixed positions as the train passes. Source data may be obtained experimentally from component tests, for example on fans or motors, or may be obtained from other prediction models, such as TWINS for rolling noise. Account should be taken of ground effects and source directivity. For a moving train the Doppler effect should also be included. For sources such as fans the effect of their installation in the train may be significant due to shielding, diffraction, or changes in airflow leading to changes in source power. A major challenge of ACOUTRAIN is to produce models that are sufficiently robust to include these effects, while also being sufficiently validated and controlled to allow their use in a regulatory situation. A key aspect is the need to account for variability, both in the source definition (e.g. one fan may differ from another) and also in the measurements.[32] Figure 7 shows a schematic view of a train and its model within the Acoutrain software, indicating the various sources and receivers used for stationary tests.
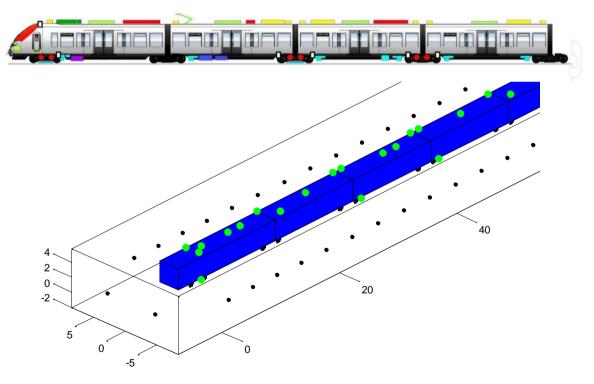

Figure 7. Upper: example of a train with various roof-mounted sources (HVAC, transformer, brake resistor, converter), and under-floor sources (air compressor, traction motor and gear box, traction motor fans); lower: model in Acoutrain software showing sources (green circles), wheelsets (black circles) and receiver microphones (black dots).

3.   Aerodynamic noise

Although rolling noise is the dominant source in many situations, aerodynamic noise increases more rapidly with speed and becomes dominant at sufficiently high speeds, as indicated in Figure 8. This shows measured data from a number of high speed trains, together with a line of slope $30 \log_{10} V$ representing rolling noise and a line of slope $80 \log_{10} V$ representing aerodynamic noise; their sum appears to give a good fit to the measured data. This gives a transition speed, at which the two components are equal, of around 370 km/h. However, a range of slopes for aerodynamic noise can still give a reasonable fit to the measured data and would imply different transition speeds.[1]

The main sources of aerodynamic noise include the front of the train, the bogie regions, the inter-coach gaps and the pantographs. The bogie region is much more important for the overall noise emission from a train than the pantograph, as there are many more bogies than pantographs. However, the pantograph can become particularly important where noise barriers are present, as elevated sources such as the pantograph may not be shielded. Research into these sources follows two routes: experimental, including wind tunnel testing, and computational.
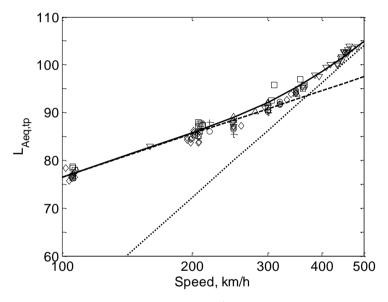

Figure 8. $L_{\text{Aeq,tp}}$ measured at 25 m from TGV-A (∇, $\square$, $\diamond$), TGV-Duplex ($\bigcirc$, $\triangle$) and Thalys (+). $---$, line with slope of $30 \log_{10} V$ to represent rolling noise; $\cdots\cdots$, line with slope of $80 \log_{10} V$ to represent aerodynamic noise; —— sum of these two components.[1]

3.1   Numerical approach

3.1.1   Computational fluid dynamics

Numerical modelling techniques for predicting aerodynamic noise have developed rapidly in recent years but the computational resources required are considerable. There are a range of different computational fluid dynamics (CFD) techniques available, which resolve differing turbulence length scales. The number of cells, and hence the computational effort, increase dramatically as the model complexity and turbulence resolution are increased.

Most aeroacoustic problems relating to trains involve dipole-type radiators where flow over solid objects lead to fluctuating drag and lift forces. In order to resolve the flow sufficiently to enable prediction of the acoustic radiation, it is necessary to use a method that gives good resolution close to the boundaries. One such approach is DES (Detached Eddy Simulation), involving a combination of LES (Large Eddy Simulation) far from the body and RANS (Random-Averaged Navier Stokes) in the boundary layer.[33] Nevertheless, such an approach involves compromises in modelling the flow and results must be checked very carefully.

Although the small scale fluctuations associated with acoustic radiation could in principle be predicted directly using CFD, for example using DNS (Direct Numerical Simulation), in practice it is preferable and more practical to predict only large scale turbulence directly and use an approximate model for the smaller scales. An acoustic analogy is then used to predict the radiated sound resulting from this in a subsequent post-processing step. Such a hybrid approach to Computational Aeroacoustics (CAA) commonly uses the Ffowcs Williams/Hawkings method for the acoustic analogy, which is based on a surface integral over a control surface.[34]

3.1.2 Example: aerodynamic noise from bogie region

As an example of a CFD/CAA approach, results are given of predictions for the bogie region of a train.[35] Initially a single simplified wheelset was modelled at 1:10 scale using DDES[33]. This was then extended to include a second wheelset and then a simplified bogie frame. Finally the bogie has been installed in a cavity under a vehicle. The computational effort becomes very large as the problem size increases, as indicated in Table 1.

Table 1. Model size and computational effort for 1:10 scale wheelsets and bogie.

Numerical cases	cells	processors	computation time (hours)
Wheelset	5.3 M	96	475
Tandem wheelsets	11.6 M	120	1100
Simplified bogie	14.5 M	160	975
Bogie in the cavity	28.1 M	260	700

Some example results are shown in Figure 9 and 10. Figure 9 visualizes vortical structures of the turbulent flow around the simplified bogie, showing how the turbulent flow from the leading wheelset impinges on the second wheelset.
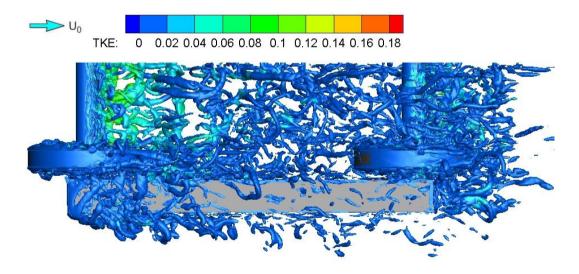

Figure 9. Iso-surface of the instantaneous normalized Q criterion coloured by turbulent kinetic energy for a 1:10 scale simplified bogie (top view), flow speed 30 m/s.[35]

Figure 10 shows predicted noise from the simplified bogie. Also shown are measurements obtained from a 1:10 scale model of a simplified bogie with the same geometry in an anechoic wind tunnel. This indicates good agreement at the main vortex shedding frequency of around 300 Hz as well as in the broadband noise. The addition of the bogie frame can be seen in Figure 10(b) to have little effect on the noise at this receiver position. Moreover, it was found that the noise from the leading wheelset is dominant at the vortex shedding frequency but the broadband contributions of both wheelsets are similar.
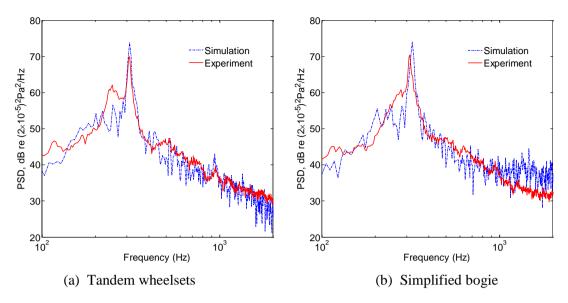
(a) Tandem wheelsets (b) Simplified bogie

Figure 10. Noise spectra at 2.5 m from a 1:10 scale simplified bogie, flow speed 30 m/s. Comparison of measurement and predictions.

3.2 Component-based approach

An alternative method of dealing with aeroacoustic problems is a semi-empirical component-based approach. This was previously developed for aircraft noise, particularly that from landing gear.[36,37] Various cylindrical components are represented by idealised spectra, which were derived from wind tunnel tests on landing gear in various stages of assembly. This provides a rapid calculation allowing parametric studies to be performed easily. However, it relies on a correct model of the component noise, and it is difficult to account for interaction effects.
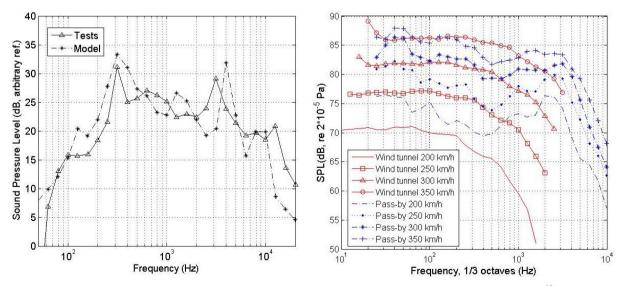

Figure 11. (a) Noise from a full scale pantograph at 330 km/h: results from wind tunnel tests[40] and component-based model.[39] (b) Comparison of measured noise spectra at 25 m from a high speed train[1] and results from wind tunnel tests on 1:10 scale bogie, converted to full scale.[39]

More recently the same approach has been applied to train components, in particular the pantograph and the bogie region.[38,39] For the pantograph, a model has been produced based on a set of cylinders, including both tonal and broadband components.[39] Figure 11(a) shows a comparison of predictions from this model with published measurements on a full scale pantograph in a wind tun-

nel.[40] For the bogie, the range of components and the associated flow field are much more complex. Nevertheless, noise spectra have been derived from measurements in a wind tunnel on a 1:10 scale bogie. After conversion to full scale these are shown in Figure 11(b) and it can be seen that they give a generally good agreement with measured noise from a high speed train below 1 kHz. Discrepancies at higher frequencies can be attributed to the dominance of rolling noise.

4. Ground vibration

Ground vibration, and the associated higher frequency problem of ground-borne noise, are increasing in importance as train speeds increase and as new underground lines are built in urban environments. Unlike airborne sound, the medium through which the vibration is transmitted differs greatly from one location to another. Moreover, as well as compressional waves, the ground can sustain shear waves, and at the ground surface Rayleigh waves are important. At a given site the soil properties also vary with depth. A major problem is therefore the identification of soil properties. No matter how complex the prediction models are, unless they are fed with the correct input parameters, the predictions will not be correct. It is often considered that models are unlikely to be able to predict vibration levels more reliably than ±10 dB.[41] Such uncertainty leads to large safety margins in the design of mitigation measures.

Apart from rare cases of trains moving at or near the wave speeds in the ground, the main excitation mechanism is due to roughness, based on similar principles as for rolling noise. A range of different modelling approaches have been used to describe ground vibration from trains; for a detailed review see [42].

4.1 Analytical models

For a homogeneous track on a halfspace ground, analytical models have been developed in which the ground response is transformed into the wavenumber domain. These can also be extended to a ground consisting of parallel soil layers[43], see Figure 12. In such a model, known as Train-induced Ground Vibration (TGV), the movement of the train is allowed for such that the roughness at one wavelength will cause a response at the corresponding frequency in the moving frame of reference but for a fixed point on the ground it will cause a response at a range of frequencies due to the Doppler effect in the ground.
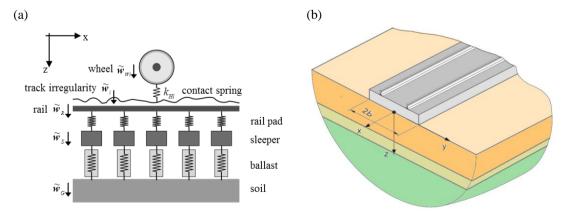

Figure 12. (a) Excitation model due to roughness and (b) the geometry of the coupled track-soil system.[44]

A simpler model treats the vehicle as stationary and allows the roughness to be 'pulled through' the contact zone between wheel and rail.[44] A further simplification involves neglecting the interaction between different wheels, by treating them as uncorrelated sources of vibration. Comparing these models, Figure 13 shows the dynamic component of rail vibration (i.e. neglecting the effect of the moving load) for a track on a homogeneous ground. In narrow-band frequency resolu-

tion there are significant differences caused by the motion of the load, but in one-third octave bands the differences are more modest.

The moving constant load also induces vibration of the track, as the rail moves up and down under each axle. However, this does not propagate into the far field unless the train speed reaches the wave speed in the ground. This quasi-static component can be predicted using the analytical model including load motion[43], but not using the 'moving roughness' model.
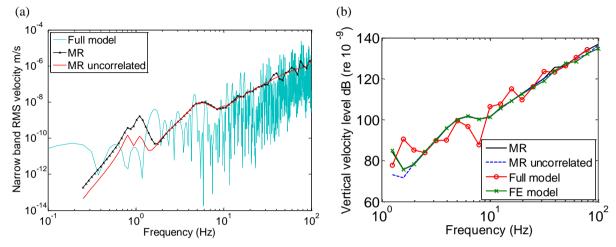

Figure 13. Dynamic component of rail velocity during passage of a train at 150 km/h. (a) Narrow band, (b) 1/3 octave band. Comparison of moving roughness (MR), moving roughness neglecting correlation between wheels, full analytical model and FE model.[44]

4.2   Wavenumber finite element/boundary element models

An alternative approach is to use numerical methods such as finite elements (FE) and boundary elements (BE) to model the ground. This has the advantage that arbitrary geometry can be considered, but the models become considerably more complex. For problems where the geometry is homogeneous in the axial direction, a wavenumber FE/BE approach can be used where the FE/BE mesh is two dimensional and the third direction is represented by a wavenumber spectrum.[45] This so-called 2.5D approach is much more efficient than a full 3D approach, while avoiding the simplifications present in a purely 2D model.

A model of a track in 2.5D is shown in Figure 14. The rails are beams while the pads, sleepers and ballast are modelled using solid elements. An example result from this approach is included in Figure 13(b) where it can be seen that agreement with the analytical result is excellent for this case. Further comparisons are given elsewhere[46], including the effects of simplifications in the vehicle model and in coupling of wheels through the rail.
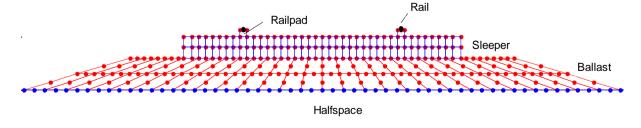

Figure 14. FE model of track included in a wavenumber FE/BE model of track/ground system.[46]

The 2.5D FE/BE approach has also been used to study various mitigation measures for ground vibration, including trenches, soft barriers and ground stiffening.[47]

Issues associated with a full 3D finite element approach are considered in [48]. Such an approach is essential if, for example, material non-linearity is to be taken into account.

4.3 A hybrid approach

In some circumstances, time domain models of vehicle/track interaction are required, for example to handle transients or non-linearities in the track or contact zone. Although a full FE model could be used, it can be more efficient to use a hybrid approach in which the track and vehicle/track interaction are modelled in the time domain and the resulting forces are applied to a separate linear model of the ground.[49] The latter may therefore be formulated in the frequency domain. Care is needed to allow for the effect of ground flexibility beneath the track in the time domain model.

Example results from such an approach are given in Figure 15. This shows the vibration caused by a train moving at different speeds at a point 8 m from the track. In Figure 15(b) the result is shown for a perfectly smooth track from which the contribution of the sleeper-passing frequency can be identified. However, it can be seen that this is much lower than the response due to track unevenness which dominates the overall response above a few Hz.
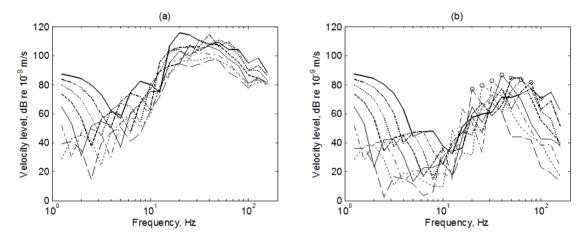

Figure 15. Ground responses at 8 m away from track for various speeds of the train: (a) total, (b) smooth track. Thinner lines: − − −, 12.5 m/s; ·····, 16 m/s; − · −, 20 m/s; —, 25 m/s; thicker lines: − − −, 32 m/s; ·····, 40 m/s; − · −, 50 m/s; —, 63 m/s. Markers indicate sleeper-passing frequency.

5. Concluding remarks

Although railways will never be silent, it is clearly important to reduce their noise and vibration as much as possible while not adding unnecessary cost or complication to their construction and operation. To achieve this it is essential to understand the sources of noise and vibration and the parameters that can influence them in order to propose cost-effective mitigation measures. Theoretical models are an important part of this process. It is important that such models should be of sufficient detail to cover the most relevant parameters in a reliable way, but not over-complicated in order not to lose insight. As the potential for developing ever more complex computational models increases, the use of simple models should not be overlooked.

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